In quest for the precise models for FDI

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Outline

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3. Accurate spectral reconstruction: problem statement
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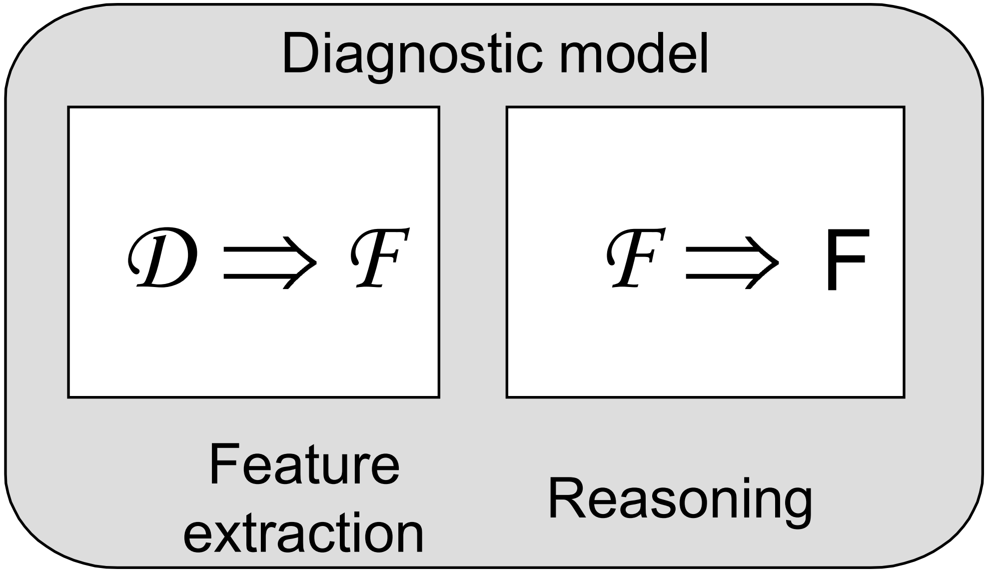
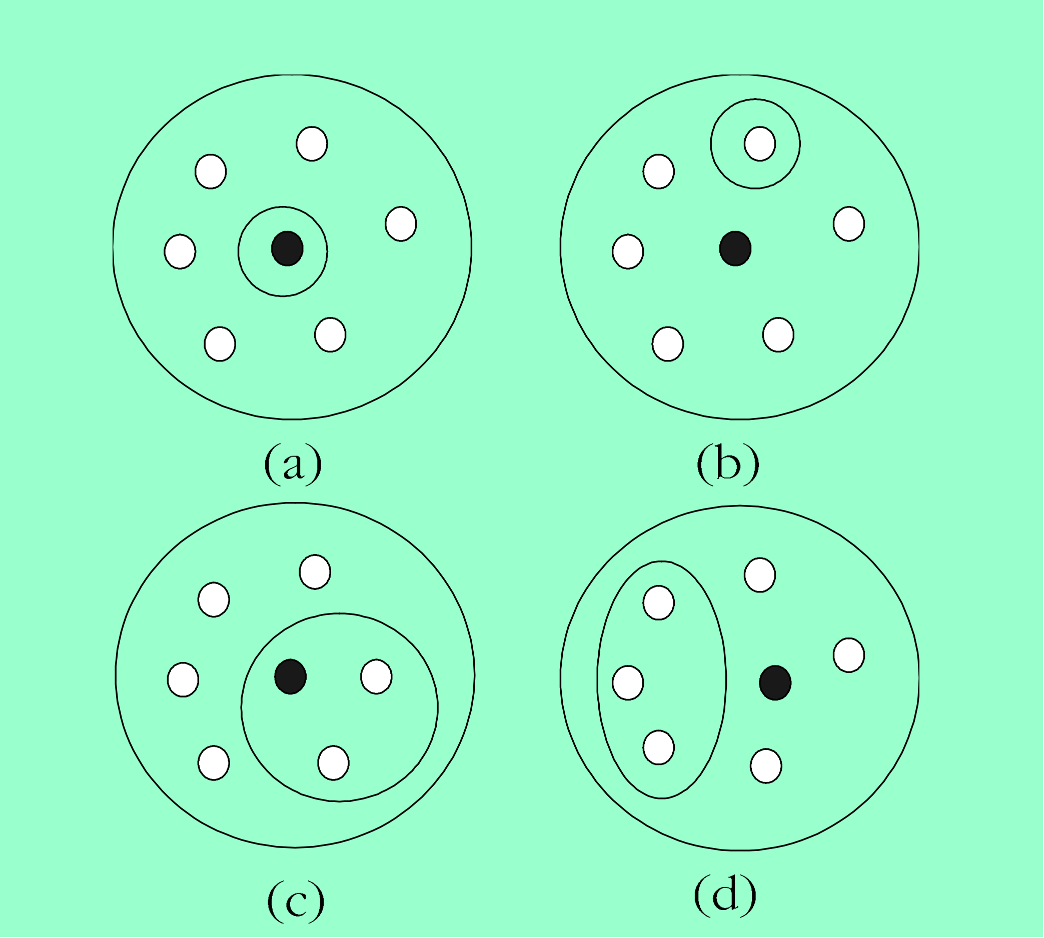
Introduction: standard diagnostic procedure



FDI performance criteria[1]

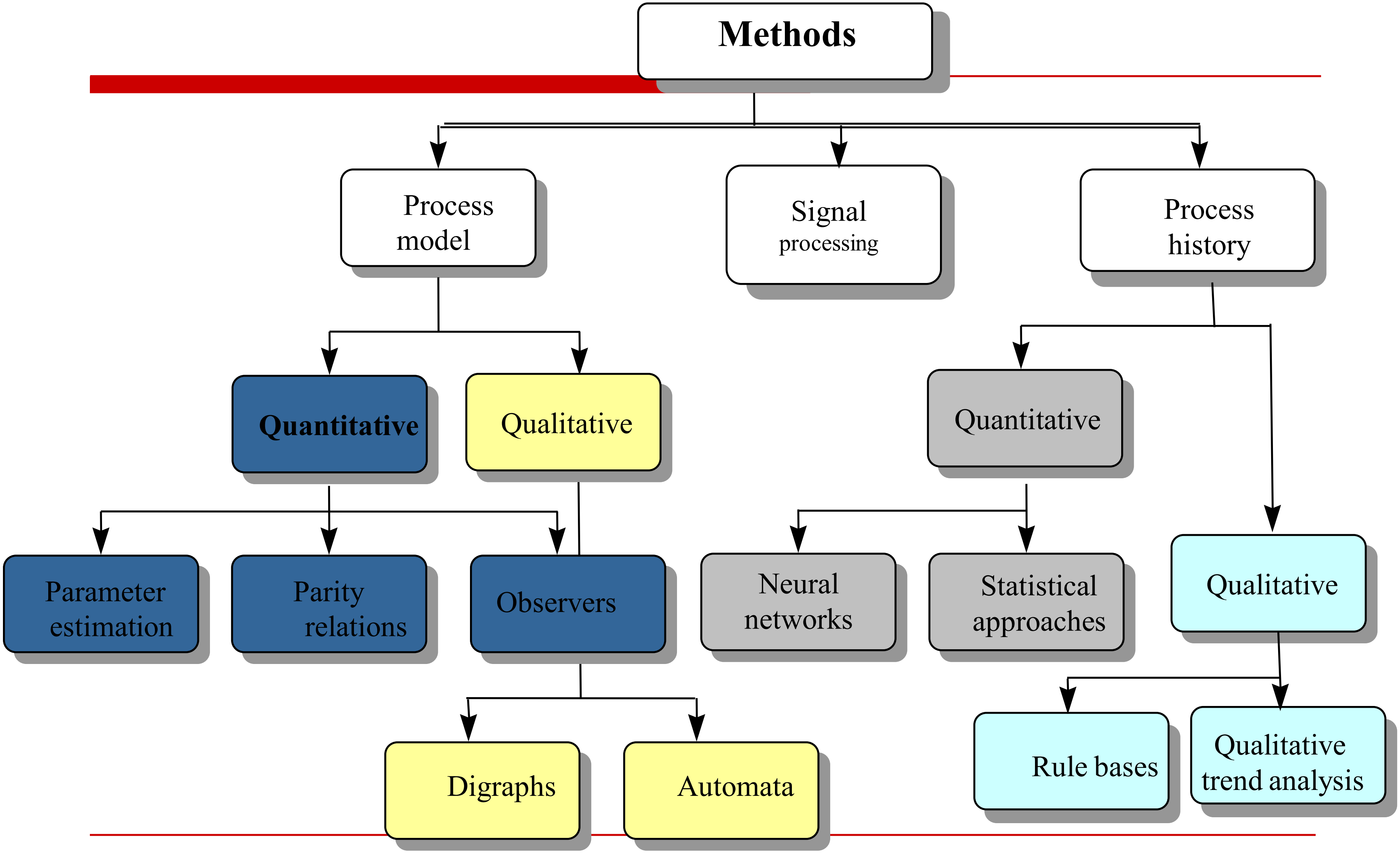
- *accuracy* : diagnosis, which includes a true fault in the set of fault candidates;
- *resolution* : the ability of the diagnostic system to provide the minimal number of fault candidates (ideally one), on the basis of the available information from the plant;
- *robustness* : accurate diagnosis in spite of modelling errors, disturbances and noise;
- *sensitivity* : the ability to detect small faults which cause only slight signal changes;
- *diagnostic stability* : stable set of fault candidates in spite of changing system excitation;
- *reliability* : accurate diagnosis for all faults including also unanticipated faults for which there is no past experience.

Illustration of precision and accuracy



	process model	measure-ments	feature extraction	reasoning
accuracy	++	+	+	+
resolution	++	+	+	+
sensitivity	++	+	+	+
robustness	++		+	+
stability			+	+
reliability	+			+

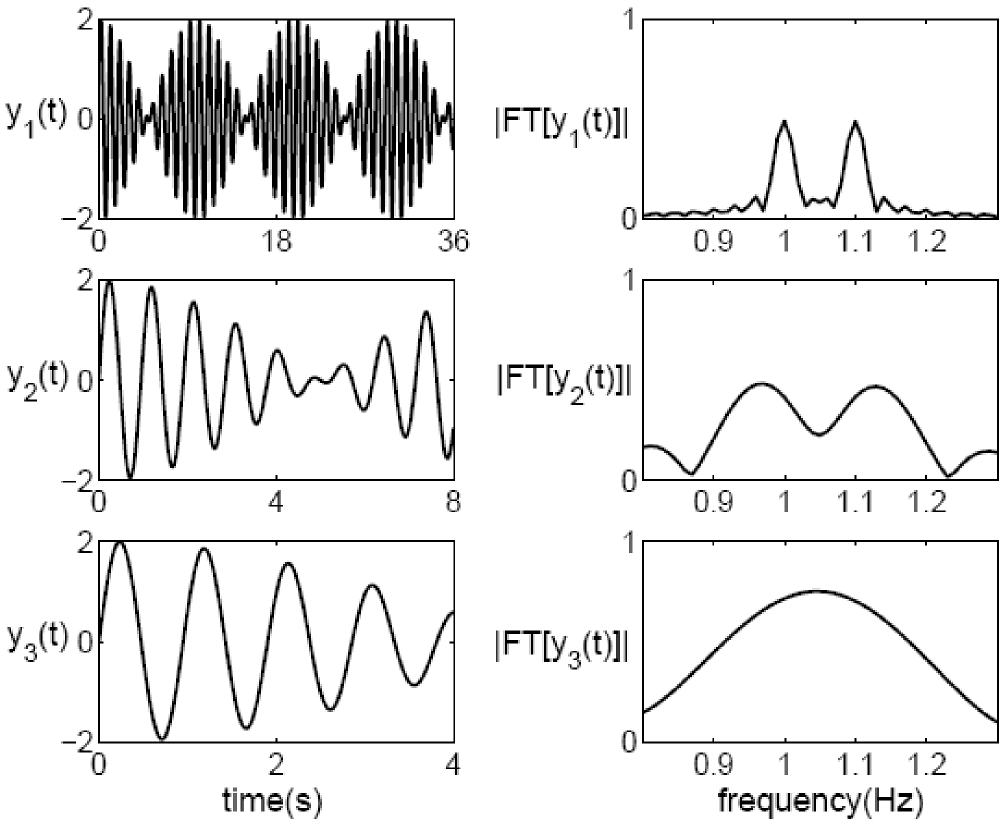
Diagnostic algorithms: overview (2)



Motivation

- Many processes are (quasi)periodic, e.g. rotational machines, biomedical applications (3)
- Repetitive phenomena over revolution cycles
- Information hidden in the frequency content
- Quality of spectral reconstruction under nonstationarities and short observation times
- link between system dynamics and signal processing?

Motivation (contd.)



Problem formulation

Gabor-Heisenberg uncertainty principle [4]

$$T^* \triangleq \int t|y(t)|^2 dt \qquad (\Delta T)^2 \triangleq \int (t - t^*)|y(t)|^2 dt$$

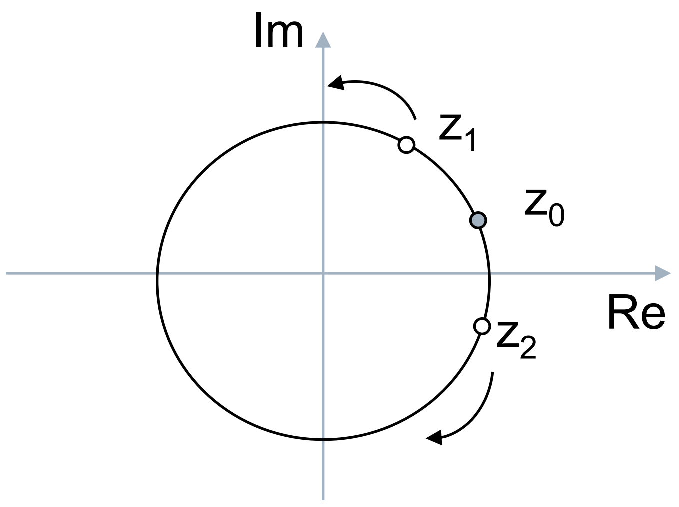
$$\omega^* \triangleq \int \omega|F(\omega)|^2 d\omega \qquad (\Delta\omega)^2 \triangleq \int (\omega - \omega^*)^2 |F(\omega)|^2 d\omega$$

$$\boxed{\Delta T \cdot \Delta\omega \geq \frac{1}{2}}$$

Problem: how to achieve small $\Delta\omega$ in spite of short ΔT?

Solution outline

Think of measured signal *y(t)* as a mixture of outputs of *N* oscillators!



$$\begin{aligned} z_1(t) &= e^{-i\lambda t} z_0 \\ z_2(t) &= e^{i\lambda t} z_0 \\ y(t) &= \overline{z}_0 z_1(t) + \overline{z}_0 z_2(t) \end{aligned}$$

$$\left.\begin{aligned} \dot{z}_1(t) &= -i\lambda z_1(t) \\ \dot{z}_2(t) &= i\lambda z_2(t) \end{aligned}\right\} \quad \dot{\mathbf{z}} = \mathbf{\Lambda z}$$

Solution outline (contd.)

More generally

$$\mathbf{x} = \mathbf{V}\mathbf{z}$$
$$\dot{\mathbf{x}} = -i\underbrace{\mathbf{V}\Lambda\mathbf{V}^{\mathrm{T}}}\mathbf{x} = -i\mathbf{A}\mathbf{x}$$
$$\mathbf{x}(t) = \mathbf{V}e^{i\Omega t}\mathbf{d} = \sum_{n=1}^{2N} d_n e^{i\omega_n t}\mathbf{v}_n$$

- N oscillators $\Rightarrow$ $2N$ states $x_1, x_2, \ldots, x_N, x_{N+1}, \ldots, x_{2N}$
- $\boldsymbol{A} = \boldsymbol{A}^T$
- $\lambda_i = -\lambda_{N+i}$

Measured signal

$$y(t) = \overline{\mathbf{x}}(0)^T \mathbf{x}(t)$$

Objective: find the eigenvalues $\lambda_1, \ldots, \lambda_N$ given the samples

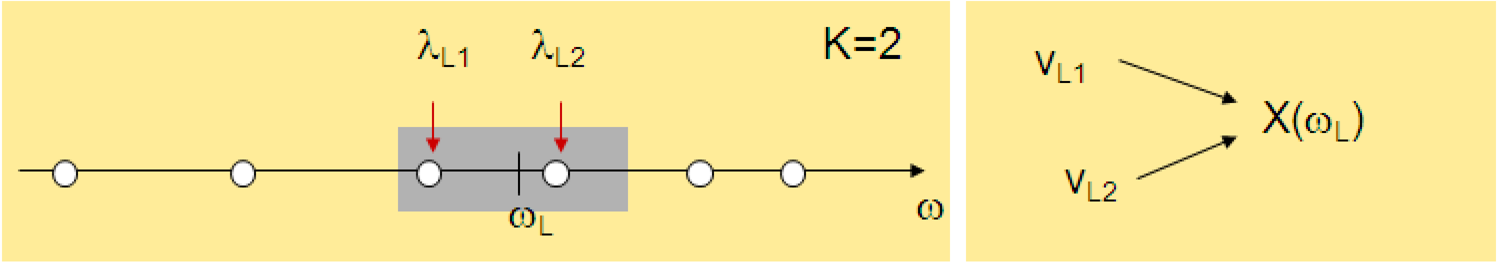
$\{y(t_0), y(t_0+\Delta t), \ldots, y(t_0+M_s\Delta t)\}$

Solution outline (contd.)

The Fourier Transform of x at some ω_L

$$
\begin{aligned}
\mathbf{X}(\omega_L) &= \int_{-\infty}^{\infty} w(t)\, e^{-i\omega_L t}\mathbf{x}(t)dt = \\
&= T\sqrt{2\pi}\sum_{n=1}^{2N} d_n e^{-(\omega_L-\lambda_n)^2\frac{T^2}{2}}\mathbf{v}_n = \\
&\approx T\sqrt{2\pi}\sum_{n=1}^{K} d_{L_n} e^{-(\omega_L-\lambda_{L_n})^2\frac{T^2}{2}}\mathbf{v}_{L_n}
\end{aligned}
$$

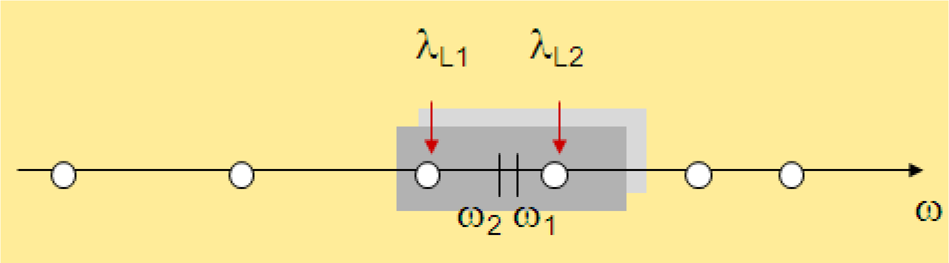
$$w(t) = e^{-t^2/2T^2}$$

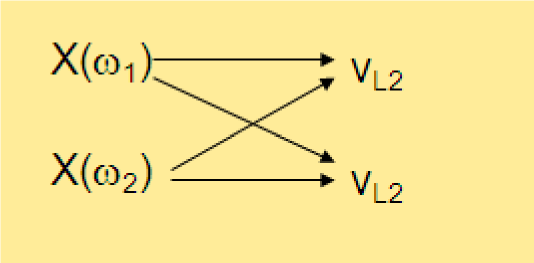


Solution outline (contd.)

How to determine v_{L1} and v_{L2}?

Take some ω_1 and ω_2 close to ω_L



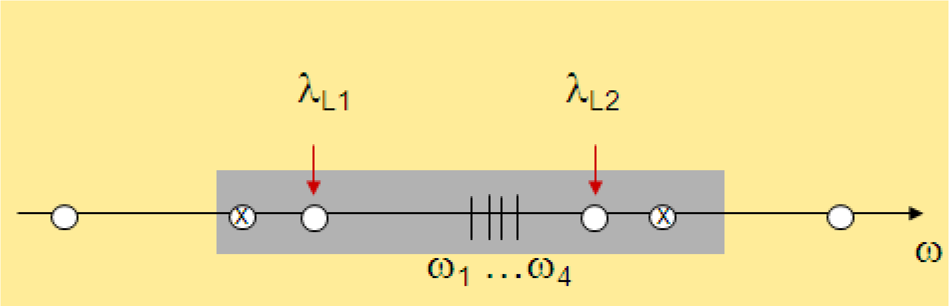


Problem: we do not know K a priori!

Solution outline (contd.)

Remedy: assume there are M eigenvalues, M>K, e.g. M=4

Consequence: now we find M(=4) eigenvalues, but only 2 are real while 2 are spurious



Solution outline (contd.)

The resulting eigenvalues (real and spurious) are calculated by the generalized eigenvalue problem (5,6)

$$\mathbf{HB} = \mathbf{SB}\mathbf{\Lambda}_{\mathrm{AV}}$$

B - matrix of generalized eigenvalues

Λ_{AV} - diagonal matrix of generalized eigenvalues

H and **S** – calculated from measured data

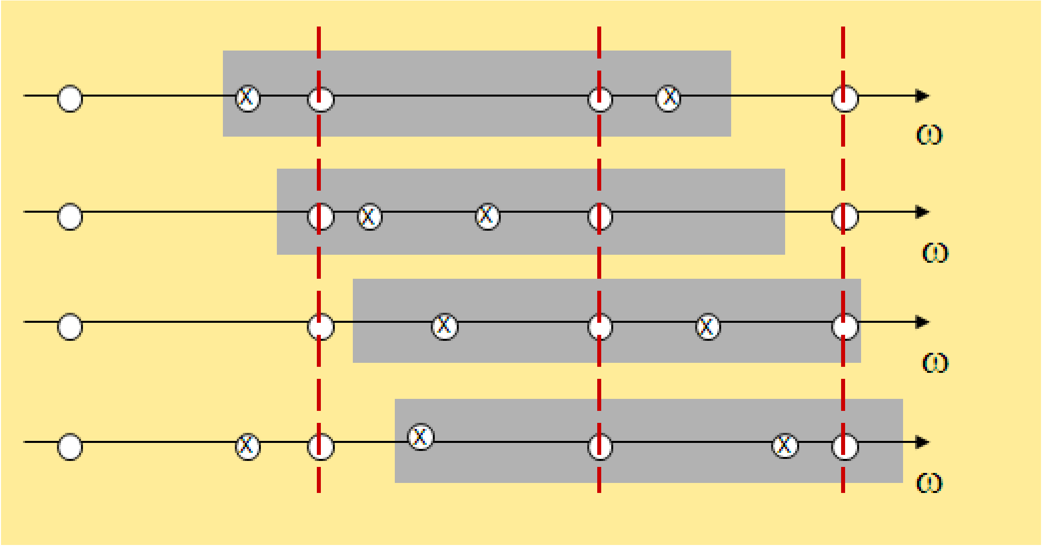
$$s_{mn} = T\sqrt{\pi}e^{-\frac{T^2(\omega_m-\omega_n)^2}{4}}\int_{-\infty}^{\infty} y(t)e^{-\frac{t^2}{4T^2}}e^{\frac{it}{2}(\omega_m+\omega_n)}dt$$

$$h_{mn} = \frac{T\sqrt{\pi}}{2}e^{-\frac{T^2(\omega_m-\omega_n)^2}{4}}\int_{-\infty}^{\infty} y(t)[\omega_m + \omega_n + i\frac{t}{T^2}]\times e^{-\frac{t^2}{4T^2}}e^{\frac{it}{2}(\omega_m+\omega_n)}dt$$

Solution outline (contd.)

Question: how to get rid of spurious eigenvalues?

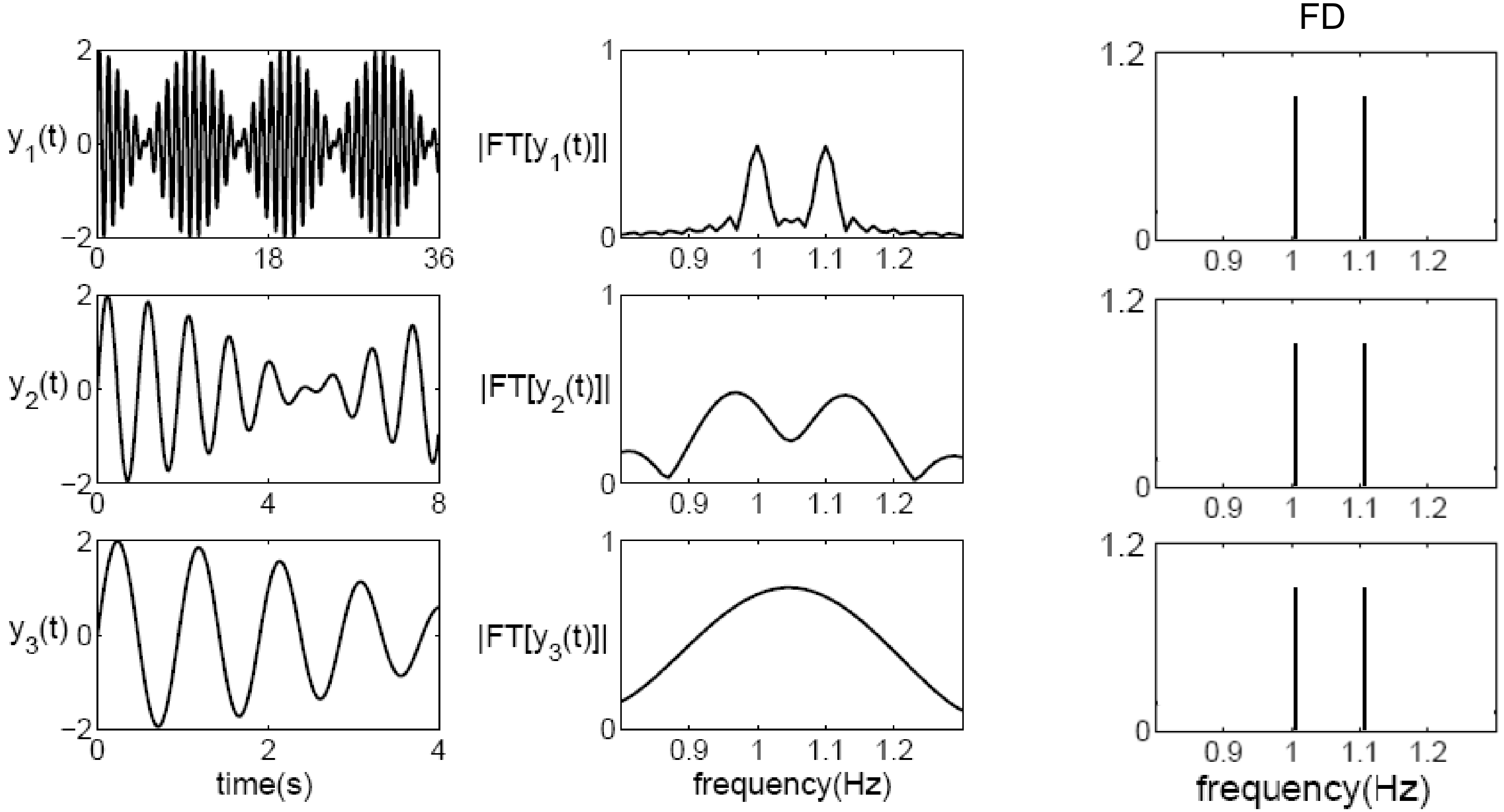
Answer: divide and conquer$\rightarrow$ Filter Diagonalization (FD)[5]



Example 1

$$y(t) = sin(2\pi f_1 t) + sin(2\pi f_2 t)$$

f_1=1Hz, f_2=1.1Hz



Influence of the measurement noise

$$y(t) = \sum_{n=1}^{2N} d_n^2 e^{i\lambda_n t} + \xi(t) \qquad \xi(t) = \begin{cases} \text{stationary} \\ E[\xi(t)] = 0, \\ E[\xi^2(t)] < \infty\,. \end{cases}$$

$$\widetilde{\mathbf{H}}\mathbf{B} = \widetilde{\mathbf{S}}\mathbf{B}\widetilde{\mathbf{\Lambda}}$$

$$\widetilde{\mathbf{H}} = \mathbf{H} + \Delta\mathbf{H}$$

$$\widetilde{\mathbf{S}} = \mathbf{S} + \Delta\mathbf{S}$$

$$\lambda \longrightarrow \lambda + \Delta\lambda$$

$$\Delta s_{mn} = T\sqrt{\pi} e^{-\frac{T^2(\omega_m-\omega_n)^2}{4}} \int_{-\infty}^{\infty} \xi(t) e^{-\frac{t^2}{4T^2}} e^{-\frac{it}{2}(\omega_m+\omega_n)} dt.$$

$$\Delta h_{mn} = \frac{T\sqrt{\pi}}{2} e^{-\frac{T^2(\omega_m-\omega_n)^2}{4}} \int_{-\infty}^{\infty} \xi(t)[\omega_m + \omega_n - i\frac{t}{T^2}] \times e^{-\frac{t^2}{4T^2}} e^{-\frac{it}{2}(\omega_m+\omega_n)} dt.$$

$$\Delta_{mn} = \int_{-\infty}^{\infty} \xi(t)\alpha(t, T, \omega_m, \omega_n)dt \qquad \alpha(t, T, \omega_m, \omega_n) \in \mathbb{C}$$

$$\Delta_{mn} = \lim_{\Delta t \to 0} \Delta t \sum_{k=0}^{N} \alpha(k\Delta t)\xi(k\Delta t)$$

For the noise term with the properties

$$|\xi| \leq M < \infty, E[\xi] = 0, E[\xi^2] = \sigma^2.$$

it holds[(6)]

$$\Delta_{mn} = \lim_{N \to \infty} \sum_{k=0}^{N} \Delta t\, \alpha(k\Delta t)\xi(k\Delta t) \Rightarrow \mathcal{N}(0, \sigma_{\Delta mn}{}^2)$$

Spectral reconstruction under noise

Analytical approach[6]

$$\Delta s_{mn} \text{ in } \Delta h_{mn}$$

$$\Downarrow$$

$$\lambda \longrightarrow \lambda + \Delta\lambda$$

$$\mathbf{HB} = \mathbf{SB\Lambda}$$

$$\mathbf{D} = \mathbf{S}^{-1}\mathbf{H} \qquad \mathbf{DB} = \mathbf{B\Lambda}$$

$$\mathbf{D} + \Delta\mathbf{D} = (\mathbf{S} + \Delta\mathbf{S})^{-1}(\mathbf{H} + \Delta\mathbf{H}).$$

$$(\mathbf{S} + \Delta\mathbf{S})^{-1} = \sum_{k=0}^{\infty}(-1)^k(\mathbf{S}^{-1}\Delta\mathbf{S})^k\mathbf{S}^{-1}$$

$$\Delta\mathbf{D} \cong -\mathbf{S}^{-1}\Delta\mathbf{S}\mathbf{S}^{-1}\mathbf{H} + \mathbf{S}^{-1}\Delta\mathbf{H}$$

$$|\Delta\lambda_i| \le \frac{\|\Delta\mathbf{D}\|_2}{s(\lambda_i)} \qquad s(\lambda) = \mathbf{y}^H\mathbf{x}$$

Conservative estimate!

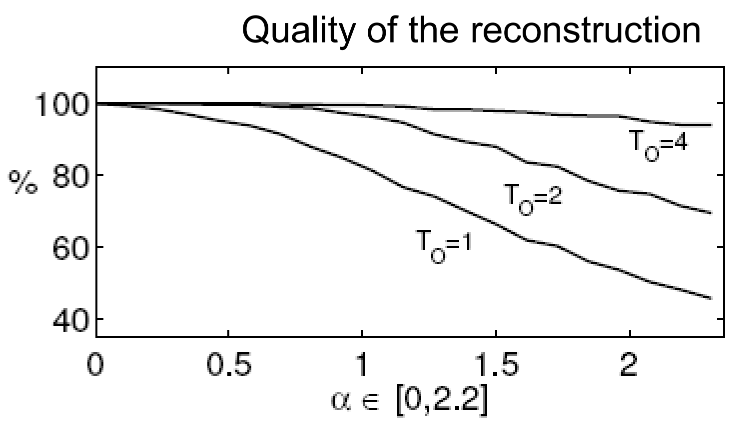
FD under noise: a simulation study on a mono-component signal

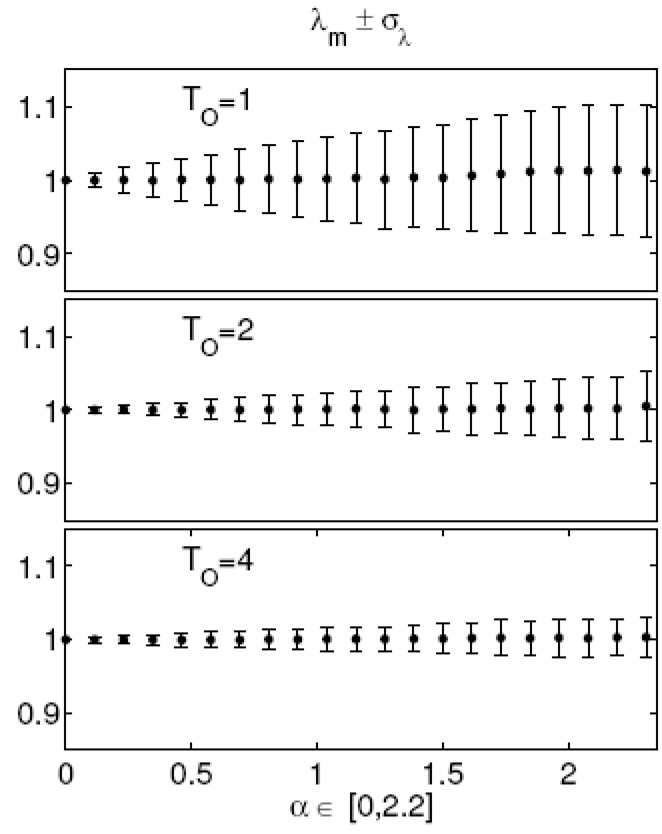
The influence of the following design parameters will be analysed[6]:

- signal length T_0
- number of samples N
- width of the Gaussian filter (T)
- auxiliary matrix dimension M

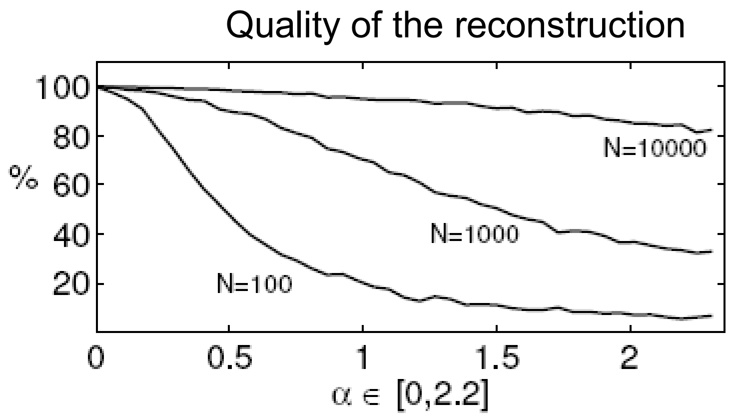
$$y(t) = sin(2\pi t) + \alpha\, \xi_{\mathcal{N}(0,1)}(t), t \in [0, T_O] \quad \alpha \in [0, 2.2]$$

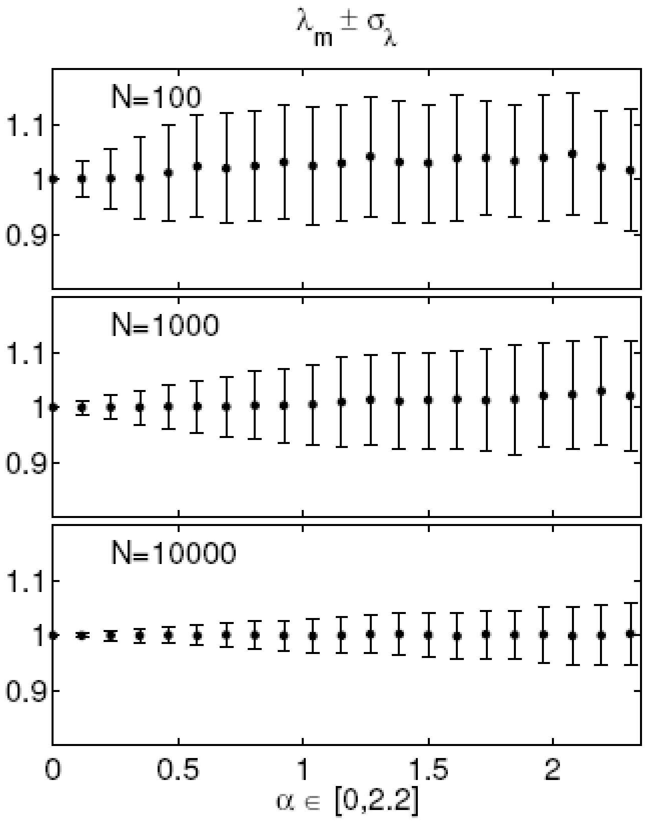
The influence of signal length T_0



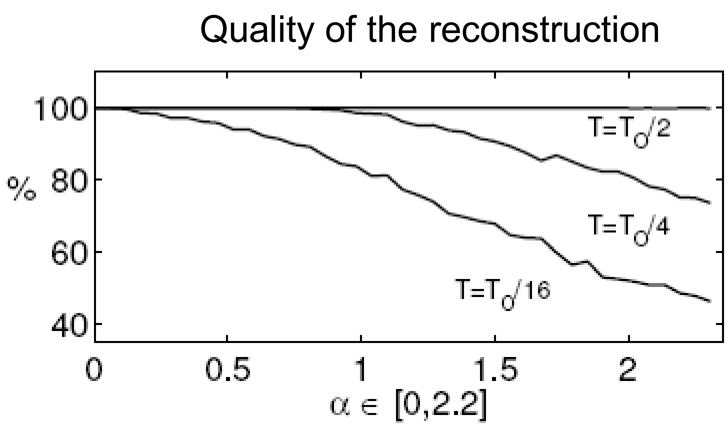


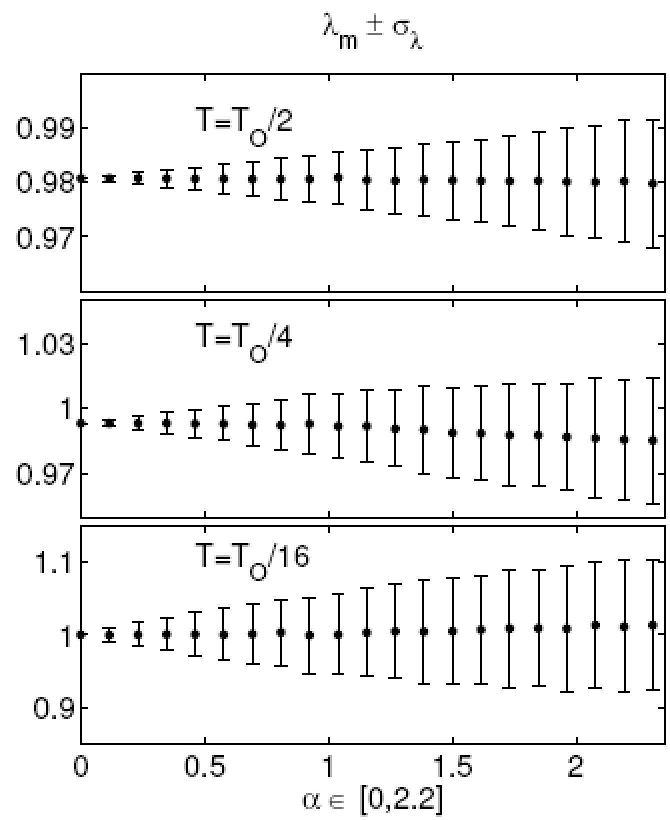
Number of samples N



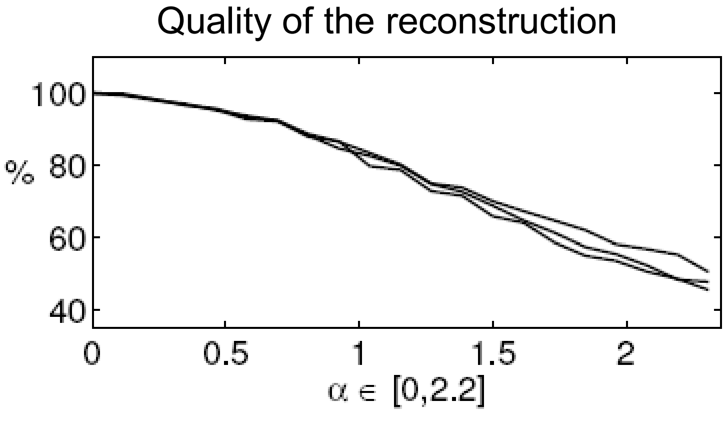


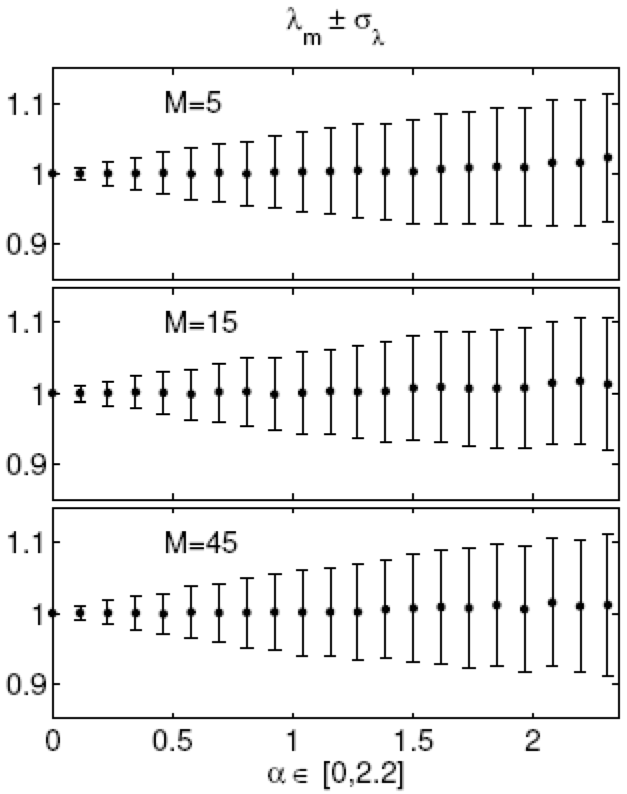
Width of the Gaussian filter T



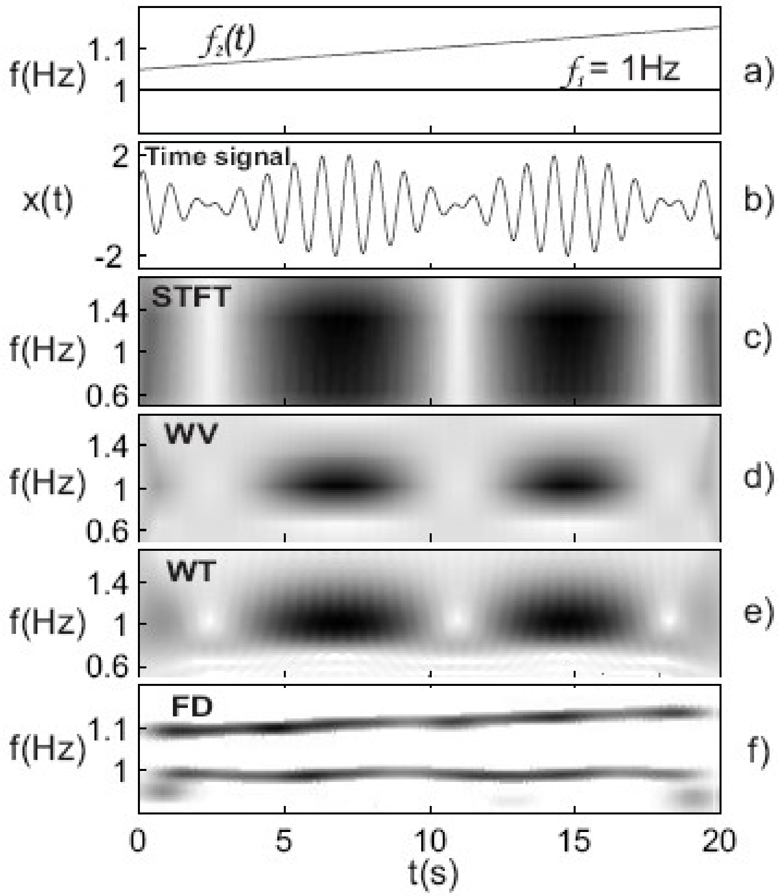


Auxiliary matrix dimension M

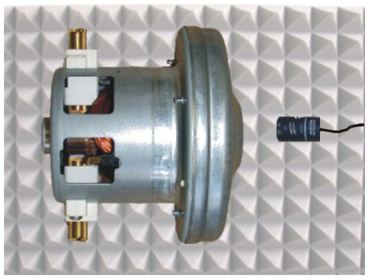


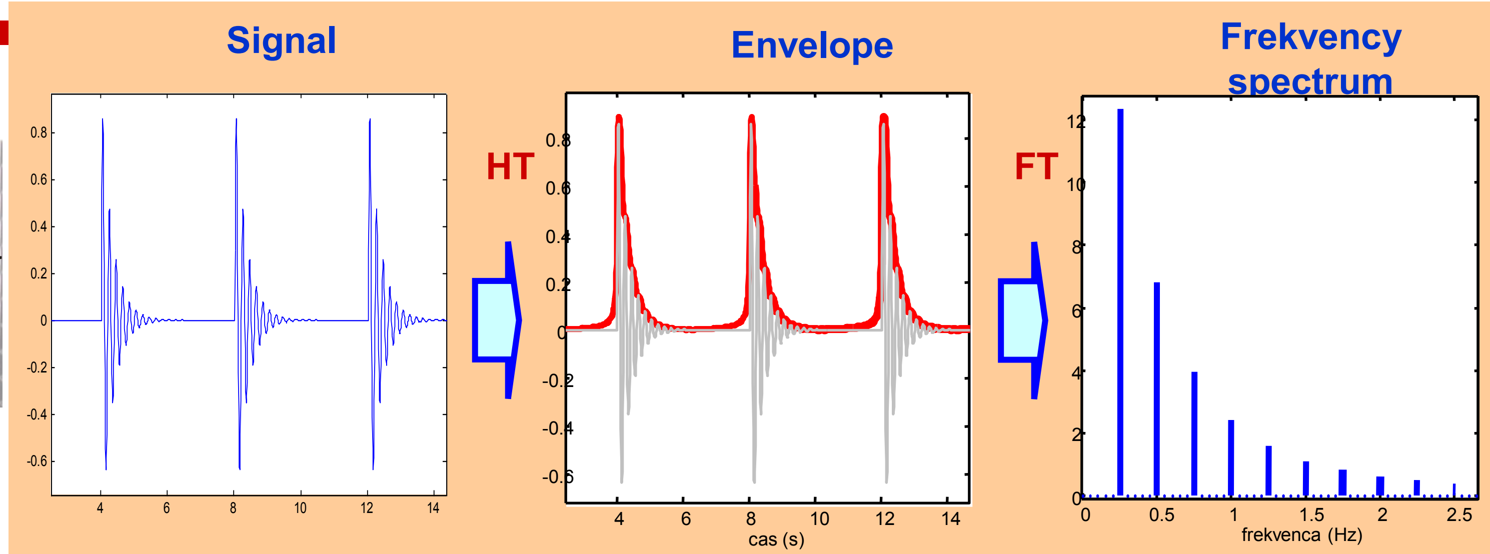


Simulation on a two-component signal

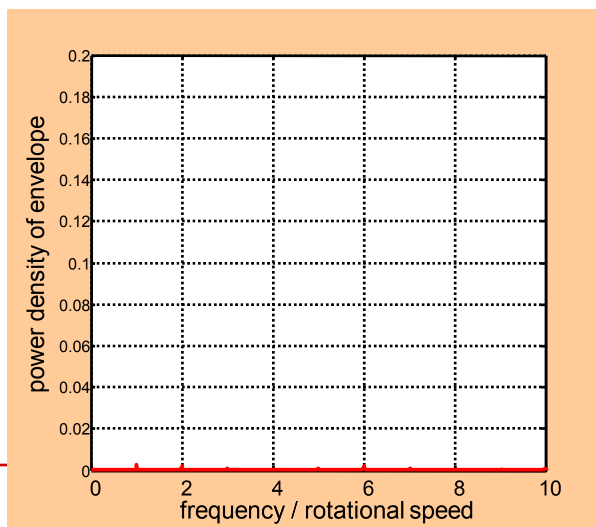


Application to the electrical motor(7)





Good



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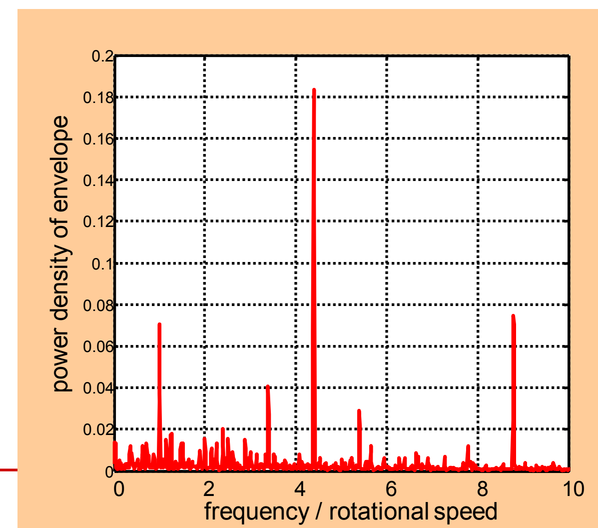
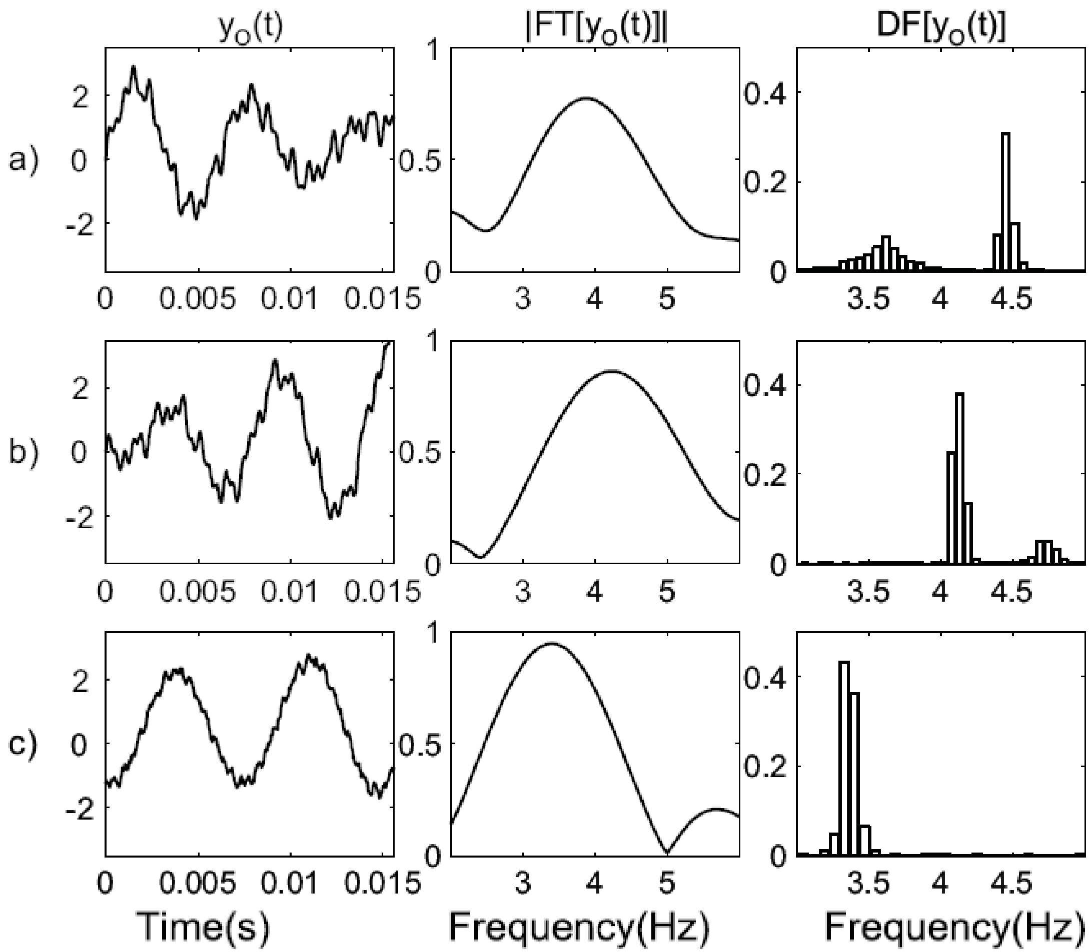


Application to the electrical motor



Conclusions

1. The technique of filter diagonalization is formulated in a system dynamics framework

2. Avoids constraints posed by Gabor-Heisenberg uncertainty limits

3. Illustrative examples clearly show benefits of FD: high resolution FDI, robust to noise

4. Mild computational load

References

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