Block independency, admissibility and decoupling: more connexions between transfer function, matrix pencil and geometric approaches

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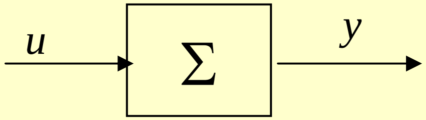
Content

Linear time-invariant system:



<u>Control objectives</u>: i/o decoupling (non interaction)

$$\Sigma : \begin{cases} \dot{x}(t) = Ax(t) + Bu(t), \\ y(t) = Cx(t); \end{cases}$$

$x(t)$: state
$u(t)$: control input
$y(t)$: output to be controlled

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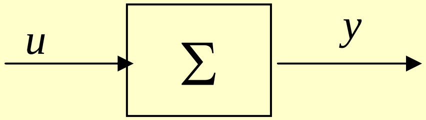

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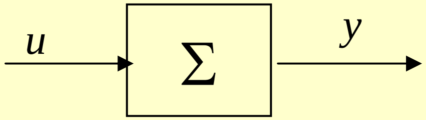

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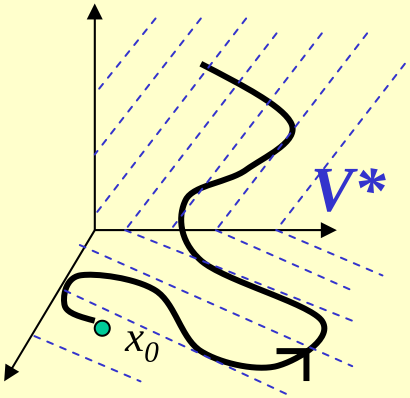
$$\mathbb{P}(s, C) = \begin{bmatrix} sI - A & -B \\ -C & 0 \end{bmatrix}$$

Geometric tools (some) (Basile and Marro, Wonham):

V* **: supremal (*A*,*B*) (or controlled)-invariant subspace in ker*C*,**

is the limit of the following non increasing algorithm:

$$\begin{cases} V_0 = X \\ V_{i+1} = \ker C \cap A^{-1}(\operatorname{Im} B + V_i) \end{cases}$$



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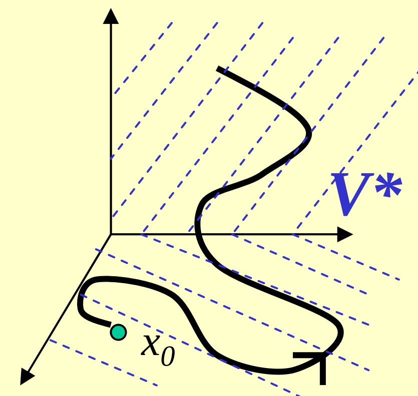
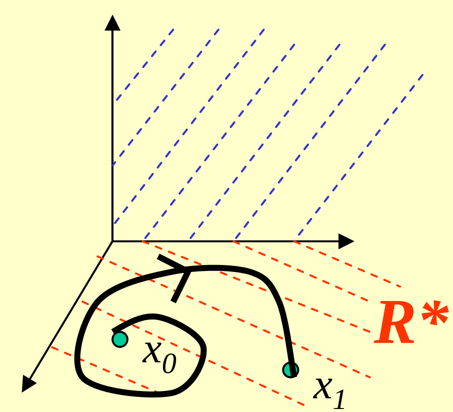
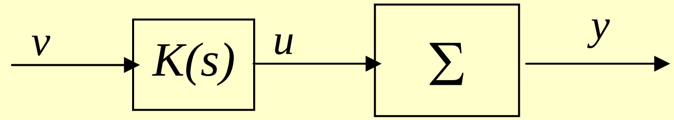


R^* : supremal (A,B) controllability subspace in ker$C$,
is the limit of the following non decreasing algorithm:

$$\begin{cases} R_0 = 0 \\ R_{i+1} = V^* \cap (AR_i + \operatorname{Im} B) \end{cases}$$



BLOCK DECOUPLING BY PRECOMPENSATION



Consider the system (A, B, C) with $m-$input channels and $p-$output channels. The output is partitioned into k blocks through the list of non negative integers $(p_1, p_2, ..., p_k)$ such that $\sum_{i=1}^{k} p_i = p$. Assume that $m \geq k$. The system (A, B, C) is said to be $(p_1, p_2, ..., p_k)$ decoupled if there exist positive integers $m_1, m_2, ..., m_k$ satisfying $\sum_{i=1}^{k} m_i = m$, such that $T(s)$ has the block diagonal form

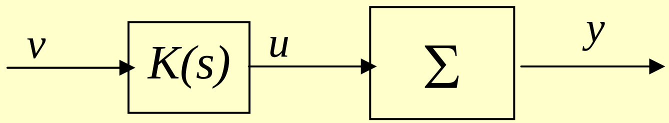
$$T(s) = \begin{bmatrix} T_{11}(s) & \dots & 0 \\ & \ddots & \\ 0 & & T_{kk}(s) \end{bmatrix}$$

$\sum_{i=1}^{k} p_i = p$ $p-$output channels

$\sum_{i=1}^{k} m_i = m$ $m-$input channels

$T_{ii}(s)$ is of dimension $p_i \times m_i$.

BLOCK DECOUPLING BY PRECOMPENSATION



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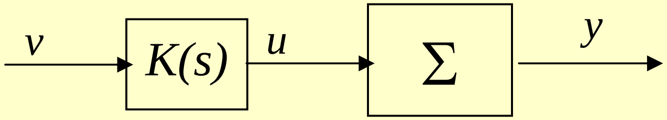
where $T_{ii}(s)$ is of dimension $p_i \times m_i$.

Control law : $u(s) = K(s)v(s),$ with $K(s)$ proper.

A rational function matrix is proper if it has a finite limit when its argument goes to infinity, in the case the limit goes to zero it is strictly proper.

ADMISSIBILITY



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with $K(s)$ proper. This is the most general control law in the sense that any static or dynamic state, or measurement, feedback is equivalent to a precompensation action of the type **(*).** But there also exist general proper precompensators which are not of feedback type. In order to exclude trivial compensators such as $K(s) = 0$, one has to add some output controllability requirements.

Hautus and Heymann introduced the notion of admissible compensators that satisfy:

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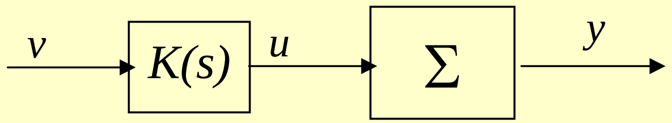

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Admissibility :

$$rank T(s)K(s) = rank T(s).$$

Hereafter $rank T(s)$ will stand for the generic rank of $T(s)$.

ROW BLOCK INDEPENDENCY

A $p \times m$ transfer function matrix is said to be partitioned in row-blocks relatively to a given list of nonnegative integers $(p_1, p_2, ..., p_k)$ s.t. $p_1 + p_2 + ... + p_k = p$ if $T(s)$ is decomposed as follows,

$$T(s) = \begin{bmatrix} T_1'(s) & \cdots & T_k'(s) \end{bmatrix}',$$

with $T_i(s)$ being $p_i \times m$ (for $i = 1, ..., k$) and $T_i'(s)$ denoting transposition of $T_i(s)$.

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Definition 1: The k row blocks of $T(s)$ are called independent if

$$rankT(s) = \sum_{i=1}^{k} rankT_i(s).$$

ROW BLOCK INDEPENDENCY

$$T(s) = \left[\begin{array}{ccc} T_1'(s) & \cdots & T_k'(s) \end{array} \right]', \qquad (\#)$$

In the sequel, we shall need certain matrices, denoted by $\overline{T}_i(s)$, deduced from matrix $T(s)$ by keeping all the row-blocks except the i-th row-block, that is to say

$$\overline{T}_i(s) = \left[\begin{array}{cccccc} T_1'(s) & .. & T_{i-1}'(s) & T_{i+1}'(s) & .. & T_k'(s) \end{array} \right]'.$$

We shall start by considering the transfer function matrix $T(s)$ as a map acting from an input linear rational space $\mathcal{U}(s)$ to an output linear rational space $\mathcal{Y}(s)$. Suppose matrix $T(s)$ is row-block partitioned as in (#) according to a given nonnegative list of integers $(p_1, p_2, ..., p_k)$. Let $\mathcal{S}_i(s)$ be the rational linear space spanned by the rows of block $T_i(s)$, for $i = 1, ..., k$.

ROW BLOCK INDEPENDENCY

$$T(s) = \begin{bmatrix} T_1'(s) & \cdots & T_k'(s) \end{bmatrix}',$$

$$\overline{T}_i(s) = \begin{bmatrix} T_1'(s) & .. & T_{i-1}'(s) & T_{i+1}'(s) & .. & T_k'(s) \end{bmatrix}'.$$

$\mathcal{S}_i(s) =$ the rational linear space spanned by the rows of block $T_i(s)$

Definition 2: The radical space of subspaces $\mathcal{S}_1(s), \mathcal{S}_2(s), \ldots, \mathcal{S}_k(s)$, $\curlyvee{\mathcal{S}}(s)$, is $\curlyvee{\mathcal{S}}(s) := \bigcap_{i=1}^{k} (\sum_{j \neq i} \mathcal{S}_j(s))$.

ROW BLOCK INDEPENDENCY

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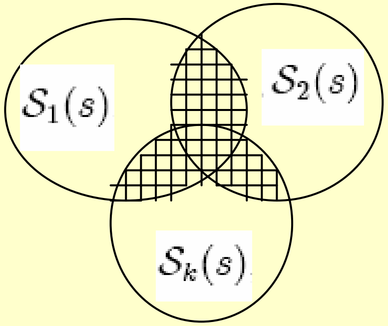

ROW BLOCK INDEPENDENCY

Proposition 2: The row block spaces $\mathcal{S}_1(s), ..., \mathcal{S}_k(s)$ are independent if and only if

$$\overset{\curlyvee}{\mathcal{S}}(s) \text{ is identically zero.}$$

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$\overset{\curlyvee}{\mathcal{S}}(s)$ is identically zero.

Proposition 3: Let $\check{\mathbf{S}}(s)$ be a strictly proper transfer function matrix such that its row span (over $R(s)$) is $\overset{\curlyvee}{\mathcal{S}}(s)$, then

$$\ker \ \check{\mathbf{S}}(s) = \sum_{i=1}^{k} \ker \overline{T}_i(s),$$

ROW BLOCK INDEPENDENCY

Corollary 4: With the previous notations, the following statements are equivalent:

1) $rank T(s) = \sum_{i=1}^{k} rank T_i(s)$.

2) $\overset{\curlyvee}{\mathcal{S}}(s) = 0$.

3) $\sum_{i=1}^{k} \ker \overline{T}_i(s) = \mathcal{U}(s)$.

4) The k row blocks of $T(s)$ are independent

MAIN RESULTS

Let us first recall the classical result of Hautus and Heymann (1983).

Theorem 5: There exists an admissible precompensator $K(s)$ such that $T(s)K(s)$ is block decoupled if and only if $\mathcal{S}_1(s), \ldots, \mathcal{S}_k(s)$ are $R(s)$ independent

[**Block independency :** $rank T(s) = \sum_{i=1}^{k} rank T_i(s) \Longleftrightarrow \check{S}(s) = 0.$]

[**Admissibility :** $rank T(s) K(s) = rank T(s).$]

Theorem 6: The following statements are equivalent,

1) There exists $K(s)$ admissible such that $T(s)K(s)$ is block decoupled,

2) $\check{S}(s) = 0$.

3) $rank T(s) = \sum_{i=1}^{k} rank T_i(s)$.

4) $\sum_{i=1}^{k} \ker \bar{T}_i(s) = \mathcal{U}(s)$, where $\mathcal{U}(s)$ is the rational input space.

5) $\sum_{i=1}^{k} \ker \mathbb{P}(s, \bar{C}_i) = \ker \begin{bmatrix} sI - A & -B \end{bmatrix}$.

6) $\sum_{i=1}^{k} \bar{\mathcal{R}}_i^* = \langle A | \; B \rangle$, where $\bar{\mathcal{R}}_i^*$ is the supremal controllability subspace in $\ker \bar{C}_i, i = 1, ..., k$ and $\langle A | \; B \rangle = \text{Im} \begin{bmatrix} B & AB & \dots & A^{n-1}B \end{bmatrix}$.

[$\bar{C}_i$ deduced from matrix C by keeping all the row-blocks except the i-th row-block.]

MAIN RESULTS : GENERAL DECOUPLING (WITHOUT ADMISSIBILITY REQUIREMENT)

Lemma 7: There exists a (general) proper precompensator, say $G_1(s)$, which block decouples $T(s)$ if and only if there exists a proper precompensator, say $G(s)$, which makes $T(s)G(s)$ decouplable by admissible precompensation.

Lemma 8: $G(s) : \mathcal{V}(s) \to \mathcal{U}(s)$ (monic) makes $T(s)G(s)$ decouplable by admissible precompensation, that is (thanks to Theorem 6) $G(s)$ satisfies, $\sum_{i=1}^{k} \ker \left[\overline{T}_i(s)G(s)\right] = \mathcal{V}(s)$ if and only if

$$\operatorname{Im} G(s) = \sum_{i=1}^{k} \ker \overline{T}_i(s).$$

MAIN RESULTS : GENERAL DECOUPLING (WITHOUT ADMISSIBILITY REQUIREMENT)

Theorem 9: Consider the k-row block structured system with transfer function matrix $T(s)$. Then $T(s)$ is decouplable by general proper precompensation if and only if,

1) $\dim(\sum_{i=1}^{k} \ker \overline{T}_i(s)) \geq k$

2) $T_i(s)\left(\sum_{i=1}^{k} \ker \overline{T}_i(s)\right) \neq 0$, for all $i = 1, ..., k$.

Example for which no admissible solution exists :

$T(s)$, with $(p_1, p_2) = (2,2)$, for which no admissible solution exists (since vector $\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$ belongs to $\mathcal{S}_1(s) \cap \mathcal{S}_2(s)$),

$$T(s) = \left[\begin{array}{c} T_1(s) \\ \hline T_2(s) \end{array}\right] = \left[\begin{array}{ccc} \frac{1}{s-1} & 0 & \frac{1}{s} \\ \frac{1}{s} & 0 & 1 \\ \hline 0 & \frac{1}{s-2} & 2 \\ 0 & \frac{s}{s-1} & \frac{1}{s} \end{array}\right],$$

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$$rank\, T_1(s) = 2$$

$$rank\, T_2(s) = 2$$

$$rank T(s) = 3.$$

$$\Longrightarrow rank T(s) \neq \sum_{i=1}^{k} rank T_i(s).$$

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let us choose $$\operatorname{Im} G(s) = \sum_{i=1}^{k} \ker \overline{T}_i(s).$$

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$$T(s) = \begin{bmatrix} \dfrac{T_1(s)}{T_2(s)} \end{bmatrix} = \left[\begin{array}{cc:c} \frac{1}{s-1} & 0 & \frac{1}{s} \\ \frac{1}{s} & 0 & 1 \\ \hline 0 & \frac{1}{s-2} & 2 \\ 0 & \frac{s}{s-1} & \frac{1}{s} \end{array}\right],$$

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let us choose $\operatorname{Im} G(s) = \sum_{i=1}^{k} \ker \overline{T}_i(s).$

Then $T(s)G(s)$ is block decoupled.

EXAMPLE

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} x(t),$$

$$T(s) = \begin{bmatrix} s^{-1} & 0 \\ s^{-2} & 0 \\ s^{-1} & s^{-2} \end{bmatrix}$$

Note that row by row decoupling is not possible (with partition $(p_1, p_2, p_3) = (1, 1, 1)$) since the total rank of $T(s)$ is $2 < p = 3$.

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Suppose one searches for block decoupling by precompensation with output partition $(p_1, p_2) = (2, 1)$.

Admissible solutions exist : $rank T(s) = \sum_{i=1}^{2} rank T_i(s)$.

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Admissible solutions exist :

$$\ker \bar{T}_1(s) = span \begin{bmatrix} s^{-1} \\ -1 \end{bmatrix}; \ \ker \bar{T}_2(s) = span \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \quad \Longrightarrow \quad \sum_{i=1}^{2} \ker \bar{T}_i(s) = \mathcal{U}(s)$$

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Admissible solutions exist :

$$\ker \begin{bmatrix} sI - A & -B \end{bmatrix} = span\left\{ v_1^T(s), \ v_2^T(s) \right\},$$

$$\ker \mathbb{P}(s, \bar{C}_1) = span\left\{ v_1(s) - s v_2(s) \right\}$$

$$\ker \mathbb{P}(s, \bar{C}_2) = span\left\{ v_2(s) \right\},$$

with $v_1'(s) = \begin{bmatrix} 1 & s & 0 & 0 & s^2 & 0 \end{bmatrix},$

$$v_2'(s) = \begin{bmatrix} 0 & 0 & 1 & s & 0 & s^2 \end{bmatrix}.$$

$$\Longrightarrow \sum_{i=1}^{k} \ker \mathbb{P}(s, \bar{C}_i) = \ker \begin{bmatrix} sI - A & -B \end{bmatrix}$$

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Admissible solutions exist :

Finally, let us denote by $\{ e_1, e_2, e_3, e_4 \}$ the canonical basis of $\mathcal{X} \approx R^4$.

$$\bar{\mathcal{R}}_2^* = \ker \bar{C}_2 = span \{ e_3, e_4 \} \text{ and}$$

$$\bar{\mathcal{R}}_1^* = \ker \bar{C}_1 = span \{ e_1, e_2 - e_3, e_4 \},$$

$$\langle A | \ B \rangle = span \{ e_2, e_4, e_1, e_3 \} = \mathcal{X}.$$

$$\Longrightarrow \quad \bar{\mathcal{R}}_1^* + \bar{\mathcal{R}}_2^* = \langle A | \ B \rangle$$

CONCLUDING REMARKS

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