POINT-MASS METHOD

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Introduction: Example I - linear non-Gaussian case:linear vs nonlinear filter (why nonlinear filtering)

$$z = x + v$$

$$\begin{aligned} p(x) &= \frac{1}{2} \qquad x \in (-1,1) \\ &= 0 \qquad \text{otherwise} \end{aligned}$$

$$\begin{aligned} p(v) &= \frac{1}{2} \qquad v \in (-1,1) \\ &= 0 \qquad \text{otherwise} \end{aligned}$$

Thus

$$E(x) = E(v) = 0$$

$$var(x) = var(v) = 1/3$$

Kalman estimate: Gaussian approximation of original pdf's,

$$p(x \mid z) = \mathcal{N}\{x : 0.5z, \frac{1}{6}\}$$

Introduction: Example I - linear non-Gaussian case:linear vs nonlinear filter (why nonlinear filtering)

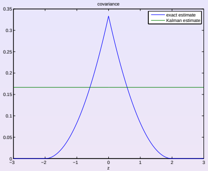
Exact solution is

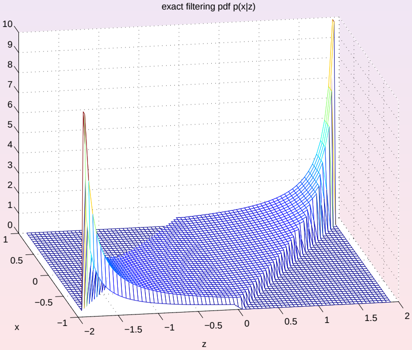
$$p(x \mid z) = \frac{[\text{sign}(1+z-x) - \text{sign}(-1+z-x)][\text{sign}(x+1) - \text{sign}(x-1)]}{2[2 - z\,\text{sign}(z)][\text{sign}(z+2) - \text{sign}(z-2)]}$$

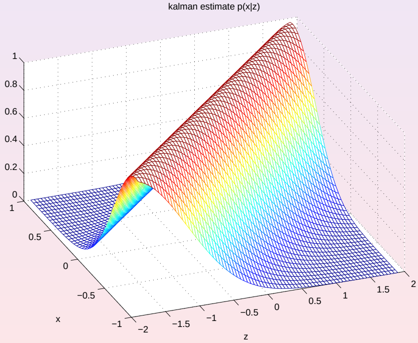
$cov(x \mid z)$ is given by

- $cov(x \mid z) = \frac{1}{3(2-z)}[1-(z-1)^3] - \frac{z^2}{4}$ for $z \in (0,2)$
- $cov(x \mid z) = \frac{1}{3(z+2)}[1+(z+1)^3] - \frac{z^2}{4}$ for $z \in (-2,0)$

for mean value $\text{E}[x|z]$ it holds that $\text{E}[x|z] = 0.5z$.


covariance
exact estimate
Kalman estimate
z


exact filtering pdf p(x|z)
x
z


kalman estimate p(x|z)
x
z

Introduction: Example II - Moment closure problem (why nonlinear filtering)

$$x_{k+1} = f_k x_k + g_k x_k^2 + w_k$$

where $\{w_k\}$ is white Gaussian process with zero mean and variance Q_k. Suppose that $\mathrm{E}[(x_k \mid z^k] = \hat{x}_k$, $\mathrm{E}[(x_k - \hat{x}_k)^2 \mid z^k] = P_k$

The aim is to compute

$$E(x_{k+1} \mid z^k) = \hat{x}'_{k+1} \quad a \quad cov(x_{k+1} \mid z^k) = P'_{k+1}$$

Let us start with mean

$$\hat{x}'_{k+1} = f_k \mathrm{E}[x_k \mid z^k] + g_k \mathrm{E}[x_k^2 \mid z^k] = f_k \hat{x}_k + g_k (P_k + \hat{x}_k^2)$$

Let us define $\tilde{x}'_{k+1} \triangleq x_{k+1} - \hat{x}'_{k+1}$, $\tilde{x}_k \triangleq x_k - \hat{x}_k$ Then

$$\tilde{x}'_{k+1} = (f_k + 2g_k \hat{x}_k)\tilde{x}_k + g_k \tilde{x}_k^2 - g_k P_k + w_k$$

$$\begin{aligned} E(\tilde{x}_{k+1}^2 \mid z^k) &= (f_k + 2g_k \hat{x}_k)^2 P_k + g_k^2 \gamma_k \\ &- g_k^2 P_k + Q_k + 2g_k (f_k + 2g_k \hat{x}_k)\delta_k \end{aligned}$$

where $\gamma \triangleq E(\tilde{x}_k^4 \mid z^k)$ $\delta_k \triangleq E[\tilde{x}_k^3 \mid z^k]$

Introduction: Stochastic system

Stochastic system

$$x_{k+1} = f_k(x_k) + w_k \quad k = 0, 1, 2, \ldots$$

- x_k is nx dimensional vector of system state at time t_k,
- w_k je nx dimensional state noise at time t, where $t_k \leq t < t_{k+1}$,
- $f_k(\cdot)$ is known vector function of proper dimension
- random process $\{w_k\}$ is white noise with known pdf $p(w_k)$
- pdf of the initial state $p(x_0)$ is known.

$$z_k = h_k(x_k) + v_k \quad k = 0, 1, 2, \ldots$$

- z_k is nz dimensional vector of known measurements at time t_k
- v_k is nz dimensional vector of measurement noise at time t_k
- random process $\{v_k\}$ is white noise with known pdf $p(v_k)$
- processes $\{w_k\}$, $\{v_k\}$ and the random variable x_0 are mutually independent.

Introduction: State estimation problem and general solution

Recursive state estimation

- Bayesian relation $p(a,b) = p(a \mid b)p(b) = p(b \mid a)p(a)$
- Filtering $p(x_k \mid z^k)$, prediction $p(x_{k+l} \mid z^k)$, smoothing $p(x_k \mid z^{k+l})$, $l > 0$
- Bayesian recursive relations

$$p(x_k \mid z^k) = \frac{p(x_k \mid z^{k-1}).p(z_k \mid x_k)}{p(z_k \mid z^{k-1})}$$

$$p(x_k \mid z^{k-1}) = \int_{-\infty}^{\infty} p(x_{k-1} \mid z^{k-1})p(x_k \mid x_{k-1})dx_{k-1}$$

$$p(z_k \mid z^{k-1}) = \int_{-\infty}^{\infty} p(x_k \mid z^{k-1})p(z_k \mid x_k)dx_k$$

- Analytical solution e.g. for linear Gaussian systems

Introduction: Point estimates

- Mean value $\hat{\mathbf{x}}_k^E$

$$\hat{\mathbf{x}}_k^E = \int_{-\infty}^{\infty} \mathbf{x}_k p(\mathbf{x}_k \mid z^k) d\mathbf{x}_k$$

- Median $\hat{\mathbf{x}}_k^{ME}$

$$\int_{-\infty}^{\hat{\mathbf{x}}_k^{ME}} p(\mathbf{x}_k \mid z^k) d\mathbf{x}_k = \int_{\hat{\mathbf{x}}_k^{ME}}^{\infty} p(\mathbf{x}_k \mid z^k) d\mathbf{x}_k$$

- Maximum aposteriori probability $\hat{\mathbf{x}}_k^{MAP}$

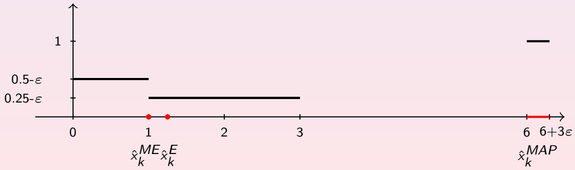
$$\hat{\mathbf{x}}_k^{MAP} = arg\ max_{\mathbf{x}_k}\ p(\mathbf{x}_k \mid z^k)$$

Introduction: Point estimates - Example 1 (why density function)

Let us calculate point estimates $\hat{x}_k^{MAP}, \hat{x}_k^{E}, \hat{x}_k^{ME}$, for filtering pdf $p(x_k \mid z^k)$ given by

$$
\begin{aligned}
p(x_k \mid z^k) &= 0,5 - \varepsilon & x_k &\in < 0, 1) \\
&= 0,25 - \varepsilon & x_k &\in < 1, 3 > \\
&= 1 & x_k &\in < 6, 6 + 3\varepsilon > \quad pro\ \varepsilon \to 0
\end{aligned}
$$

- $\hat{x}_k^{MAP} \in < 6, 6 + 3\varepsilon >$
- $\hat{x}_k^{E} = 1, 25$
- $\hat{x}_k^{ME} = 1$

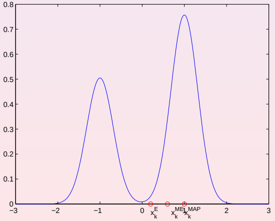


Introduction: Point estimates - Example 2 (why density function)

Let us calculate point estimates $\hat{x}_k^{MAP}, \hat{x}_k^{E}, \hat{x}_k^{ME}$, for filtering pdf $p(x_k \mid z^k)$ given as

$$p(x_k \mid z^k) = 0.4 \cdot \mathcal{N}\{x_k : -1, 0.1\} + 0.6 \cdot \mathcal{N}\{x_k : 1, 0.1\}$$

- $\hat{x}_k^{MAP} = 1$
- $\hat{x}_k^{E} = 0.2$
- $\hat{x}_k^{ME} = 0.69$

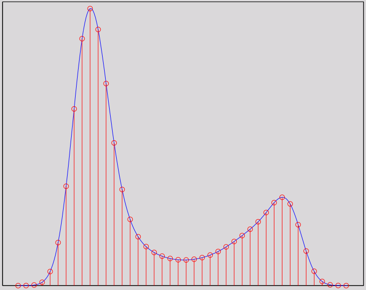


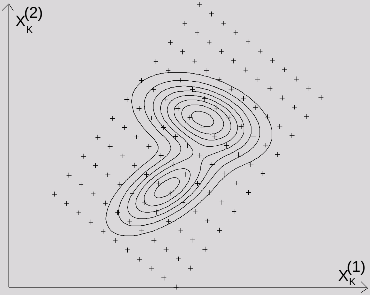
Introduction: Basic types of pdf approximation - Point-mass method

Point-mass method

$$p(\mathbf{x}_k|\mathbf{z}^k)=\{P_i;\, P_i = p(\mathbf{x}_k \in \text{neighbourhood } \xi_i|\mathbf{z}^k)\},$$

$\xi_i \in \Xi(N)$

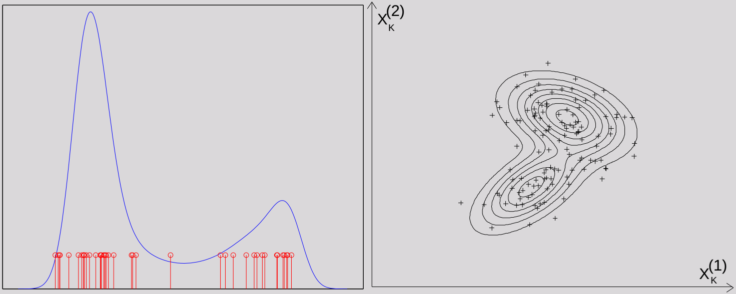




Introduction: Basic types of pdf approximation - Sequential Monte Carlo method

Sequential Monte Carlo method

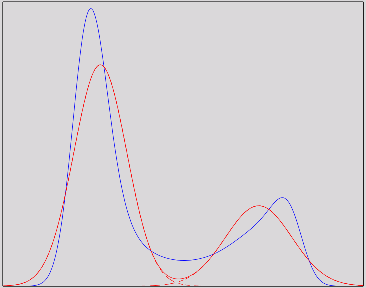
$$p(\mathbf{x}_k|\mathbf{z}^k) = \sum_{i=1}^{N} w_k^{(i)} \delta(\mathbf{x}_k - \mathbf{x}_k^{(i)})$$

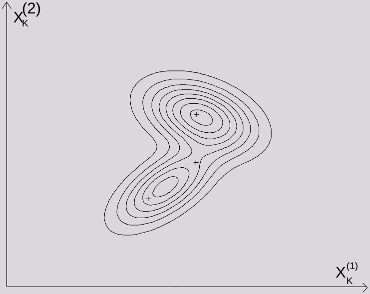


Introduction: Basic types of pdf approximation - Gaussian sum method

Gaussian sum method

$$p(\mathbf{x}_k|\mathbf{z}^k) = \sum_{i=1}^{N} \alpha_k^{(i)} \mathcal{N}\{\mathbf{x}_k : \mu_k^i, \mathbf{P}_k^i\}$$



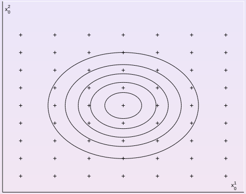


Point mass method: Development stages

1. Bucy, R. and K. Senne (1971): Digital synthesis of non-linear filters. Automatica 7(3), 287–298.
2. Kramer, S. and H.W. Sorenson (1988): Recursive Bayesian estimation using piecewise constant approximations. Automatica 24(6), 789–801.
3. Šimandl M., Královec J. , Söderström T. (2006): Advanced point mass method for nonlinear state estimation, Automatica 42, Issue 7, 1133-1145

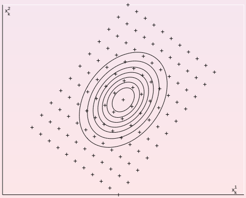
Point-mass method: Basic algorithm

Step 1: Define an initial grid for $p(\mathbf{x}_0|\mathbf{z}^{-1})$: $\Xi_0(N_0) = \{\underline{\xi}_{0i}; i = 1, 2, \ldots, N_0\}$



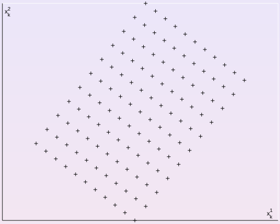
Step 2 (Measurement update) Time k: Compute $p(\mathbf{x}_k|\mathbf{z}^k)$ for $\Xi_k(N_k)$

$p(\underline{\xi}_{ki}|\mathbf{z}^k) = c_k^{-1} p(\underline{\xi}_{ki}|\mathbf{z}^{k-1})\, p_{v_k}(\mathbf{z}_k - \mathbf{h}_k(\underline{\xi}_{ki}))$ $c_k = \sum_{i=1}^{N_k} \Delta\underline{\xi}_{ki}\, p(\underline{\xi}_{ki}|\mathbf{z}^{k-1})\, p_{v_k}(\mathbf{z}_k - \mathbf{h}_k(\underline{\xi}_{ki}))$



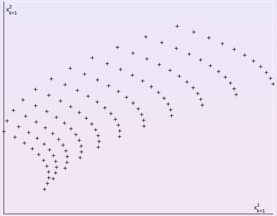
Point-mass method: Basic algorithm

Step 3: Transformation $\Xi_k(N_k) \xrightarrow{\mathbf{f}_k} H_{k+1}(N_k)$;



$\Xi_k(N_k)$

$\overset{\mathbf{f}_k}{\longmapsto}$



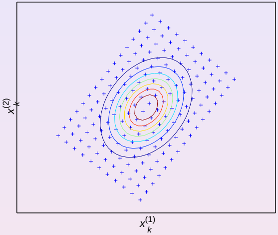
$H_{k+1}(N_k)$

Step 4: Redefine $H_{k+1}(N_k)$: $\Xi_{k+1}(N_{k+1}) = \{\underline{\xi}_{k+1,j}; j = 1, 2, \ldots, N_{k+1}\}$

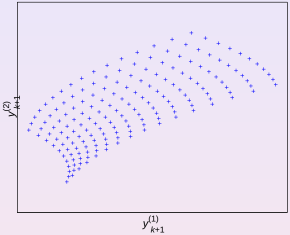
Step 5: Compute $p(\mathbf{x}_{k+1}|\mathbf{z}^k)$ for $\Xi_{k+1}(N_{k+1})$

$$p(\underline{\xi}_{k+1,j}|\mathbf{z}^k) = \sum_{i=1}^{N_k} \Delta\underline{\xi}_{ki}\, p(\underline{\xi}_{ki}|\mathbf{z}^k)\, p_{w_k}(\underline{\xi}_{k+1,j} - \underline{\eta}_{k+1,i})$$

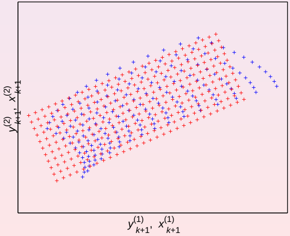
Point-mass method: Graphical illustration of basic algorithm



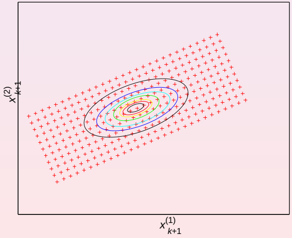
1. Filtering



2. Grid transformation



3. Grid resetting



4. Prediction

Point-mass method: Weaknesses of standard algorithm

- the setting of the number of grid points not specified
  - incomplete description of grid design
  - minimum sufficient number of grid points not specified
- enormous computational demands, especially for multimodal pdf's
  - multigrid representation
  - grid splitting
  - grid merging

Grid design: Anticipative approach

- The task is to set a suitable number N_{k+1} of grid points for the grid $\Xi_{k+1}(N_{k+1})$
- It will affect the approximation quality of the discrete convolution at the *next* time step $k+2$.
- The idea of the anticipative approach: Design of the grid is based on its future behaviour respecting characteristics of the system

The number of grid points N_{k+1} is determined by

- the length of a significant support I_{k+1} of $p(x_{k+1}|z^k)$
- by the distance $\Delta\xi_{k+1}$ of two neighbouring grid points.

Grid design: Anticipative approach (cont'd)

The convolution integral written for a single point X_{k+2}

$$p(X_{k+2}|z^{k+1}) = \int p(x_{k+1}|z^{k+1})\, p(X_{k+2}|x_{k+1})\, \mathrm{d}x_{k+1}$$

can be approximated by

$$p(X_{k+2}|z^{k+1}) \approx \Delta\xi_{k+1} \sum_{j=1}^{N_{k+1}} P_{k+1|k+1,j}\, p_{w_{k+1}}(X_{k+2} - \eta_{k+2,j})$$

where $\eta_{k+2,j} = f_{k+1}(\xi_{k+1,j})$, and $\Delta\xi_{k+1} = \Delta\xi_{k+1,j}$ for $j = 1, \ldots, N_{k+1}$,

- It is necessary to provide enough grid points $\eta_{k+2,j} \in H_{k+2}(N_{k+1})$ in the neighbourhood of the point $X_{k+2} \in I_{k+2}$ to ensure a sufficient approximation quality of the convolution.
- The size of the neighbourhood of X_{k+2} is determined by the variance of the state noise w_{k+1} because X_{k+2} can be interpreted as the mean value of the random variable s_{k+2} with a pdf defined as $p(s_{k+2}) = p_{w_{k+1}}(X_{k+2} - s_{k+2})$.

Grid design: Anticipative approach (cont'd)

A case where the support of the pdf's $p_{w_{k+1}}(X_{k+2} - s_{k+2})$ is covered by at least three points $\eta_{k+2,j} \in H_{k+2}(N_{k+1})$

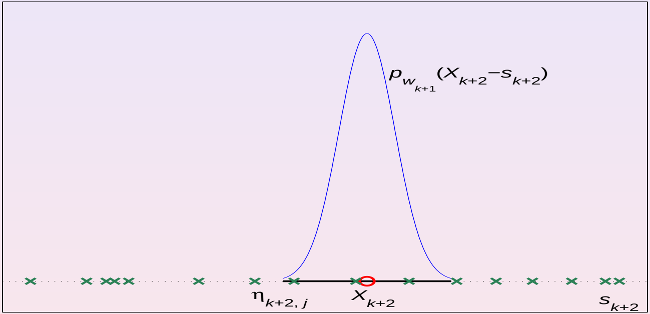


Figure: The covering of supports of the pdf's $p_{w_{k+1}}(X_{k+2} - s_{k+2})$ by grid points $\eta_{k+2,j}$. The point X_{k+2} is denoted by the circle and points $\eta_{k+2,j}$ are denoted by $\times$-marks.

Grid design: Anticipative approach (cont'd)

The “sufficiency” of the number N_{k+1} of grid points $\eta_{k+2,j}$ may be expressed by

- $a > 0$ - the length of a non-negligible support of $p_{w_{k+1}}$
- $m \in \{1, 2, 3, \dots\}$ - the covering of the support by grid points

The parameter a determines what probability P_a given by $p_{w_{k+1}}$ will be taken for non-negligible

$$P(-a\sqrt{Q_{k+1}} \leq w_{k+1} \leq a\sqrt{Q_{k+1}}) = P_a$$

The parameter m expresses the requirement that at least m grid points $\eta_{k+2,j}$ cover the significant support of $p_{w_{k+1}}$ and thus

$$(X_{k+2} - \eta_{k+2,j}) \in \left[-a\sqrt{Q_{k+1}},\, a\sqrt{Q_{k+1}}\right]$$

for any point $X_{k+2} \in I_{k+2}$.

$$\Delta\eta_{k+2,j} \leq 2\frac{a}{m}\sqrt{Q_{k+1}}\,; \quad j = 1, 2, \dots, N_{k+1}$$

Grid design: Anticipative approach (cont'd)

The condition should be modified for $\Delta\xi_{k+1,j}$ because distances for the points $\eta_{k+2,j} \in H_{k+2}(N_{k+1})$ can be set only via the grid points $\xi_{k+1,j} \in \Xi_{k+1}(N_{k+1})$. Using the well-known relation for transformation of random variables

$$p_{s_{k+2}|z^{k+1}}(\eta_{k+2,j}|z^{k+1}) = \frac{P_{k+1|k+1,j}}{\left|J_{k+1}(\xi_{k+1,j})\right|}\,; \quad j = 1, 2, \ldots, N_{k+1}$$

where $J_{k+1}(x_{k+1}) = \frac{\mathrm{d}f_{k+1}(x_{k+1})}{\mathrm{d}x_{k+1}}$, yields

$$\Delta\eta_{k+2,j} = \left|J_{k+1}(\xi_{k+1,j})\right|\Delta\xi_{k+1}$$

where the index j in $\Delta\xi_{k+1,j}$ can be omitted because this distance is assumed to be constant.

Grid design: Anticipative approach (cont'd)

Now

$$\Delta\xi_{k+1} \leq 2\frac{a}{m}\sqrt{Q_{k+1}}\left|J_{k+1}(\xi_{k+1,j})\right|^{-1}; \quad j = 1, 2, \ldots, N_{k+1}$$

Since $\xi_{k+1,j}$ are not known yet, any point of the significant support I_{k+1} of the predictive pdf $p(x_{k+1}|z^k)$ must fulfill

$$\Delta\xi_{k+1} \leq 2\frac{a}{m}\sqrt{Q_{k+1}}\left[\max_{x_{k+1}\in I_{k+1}}\left|J_{k+1}(x_{k+1})\right|\right]^{-1}$$

In case of more complicated functions $f_{k+1}(x_{k+1})$ it is possible to approximate the maximum numerically as

$$\max_{i=1,\ldots,N_k}\left|J_{k+1}(\eta_{k+1,i})\right|, \quad \eta_{k+1,i} \in H_{k+1}(N_k) .$$

The maximum value of the distance for the new grid $\Delta\xi^*_{k+1}$.

Grid design: Anticipative approach (cont'd)

The number of grid points should satisfy

$$N_{k+1} \geq \frac{\mathrm{d}(I_{k+1})}{\Delta\xi^*_{k+1}} = \frac{\mathrm{d}(I_{k+1})}{2\gamma} Q_{k+1}^{-\frac{1}{2}} \max_{x_{k+1} \in I_{k+1}} \left| J_{k+1}(x_{k+1}) \right|$$

with $\gamma = \frac{a}{m}$ and $I_{k+1} = [\hat{\eta}_{k+1} - b\sigma_{k+1}, \hat{\eta}_{k+1} + b\sigma_{k+1}]$

- Values of the parameters should satisfy the empirical conditions $b \geq 3$, $a \geq 3$, $m \geq 3$, and $\gamma \leq 1$ to ensure a sufficient quality of the estimates.
- The condition $\gamma \leq 1$ may be used independently of a and m.
- In practical implementations of the algorithm, the parameters b and γ are likely to be set constant for all instants $k = 0, 1, 2, \ldots$.

Grid design: Anticipative grid design algorithm for one-dimensional system

1. Compute estimates of the first two moments of the predictive pdf $p(x_{k+1}|z^k)$ as

$$\hat{\eta}_{k+1} = \Delta\xi_k \sum_{i=1}^{N_k} \eta_{k+1,i} P_{k,i}$$

$$\sigma_{k+1} = \Delta\xi_k \sum_{i=1}^{N_k} \eta_{k+1,i}^2 P_{k,i} - \hat{\eta}_{k+1}^2 + Q_k$$

2. For a chosen b set the non-negligible support of $p(x_{k+1}|z^k)$ as $I_{k+1} = [\hat{\eta}_{k+1} - b\sigma_{k+1}, \hat{\eta}_{k+1} + b\sigma_{k+1}]$. The length of the support is

$$\mathrm{d}(I_{k+1}) = 2b\sigma_{k+1} \ .$$

3. For a chosen γ set the number of grid points N_{k+1} satisfying

$$N_{k+1} \geq \frac{b\sigma_{k+1}}{\gamma} Q_{k+1}^{-\frac{1}{2}} \max_{x_{k+1} \in I_{k+1}} \left| J_{k+1}(x_{k+1}) \right|$$

where $J_{k+1}(x_{k+1}) = \frac{\mathrm{d} f_{k+1}(x_{k+1})}{\mathrm{d} x_{k+1}}$. The maximum of $|J_{k+1}(x_{k+1})|$ is computed analytically, if possible, or else approximated by

$$\max_{i=1,\ldots,N_k} \left| J_{k+1}(\eta_{k+1,i}) \right|, \quad \eta_{k+1,i} \in H_{k+1}(N_k) \ .$$

4. Compute the point mass $\Delta\xi_{k+1}$ using the chosen N_{k+1} as

$$\Delta\xi_{k+1} = \frac{d(I_{k+1})}{N_{k+1}}$$

5. Place grid points $\xi_{k+1,j} \in \Xi_{k+1}(N_{k+1})$ to cover the support I_{k+1}

$$\xi_{k+1,j} = \hat{\eta}_{k+1} + \Delta\xi_{k+1} \left(j - \frac{N_{k+1}+1}{2} \right)$$

Multigrid design

Multimodal pdf

representation of state space by one grid is unsuitable

- covering areas of state space with negligible probability of state presence
- high computational demands

$\Rightarrow$ introducing multigrid representation

Multigrid point-mass representation of pdf $p_{\mathbf{x}_k}(\mathbf{x}_k)$

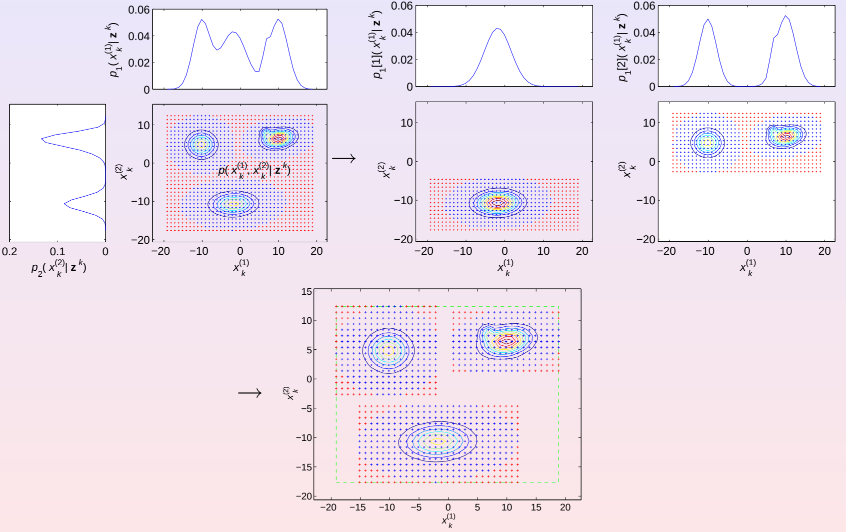
- set of grids: $\{\Xi_k[\mu](N_k[\mu]); \mu = 1, \ldots, M_k\}$
- set of pdf values: $\{\mathcal{P}_k[\mu]; \mu = 1, \ldots, M_k\}$
- grid weight: $\omega_k[\mu] = \Delta\xi_k[\mu] \sum_{i=1}^{N_k[\mu]} P_{ki}[\mu]$

Multigrid design (cont'd)

Multigrid representation requires modification of the basic algorithm and new algorithm steps

- each grid is handled separately
  - — repeated application of the basic algorithm
- each grid is evaluated by the grid weight $\omega_k[\mu]$
- grid management: splitting and merging

Multigrid design: Grid splitting using marginal pdf's

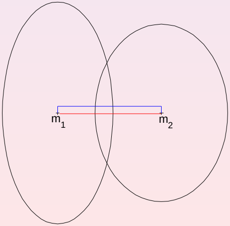


Multigrid design: Grid merging using Mahalanobis distance

- grids are merged after the time update step (prediction causes increase of uncertainty, transformation of grids may cause their overlapping)
- Mahalanobis distance decision rule: Grids are merged if one of the M-distances between grid centers is less than δ.

$$[(m_1 - m_2)^{\mathrm{T}} \mathbf{C}_1^{-1} (m_1 - m_2)]^{\frac{1}{2}} < \delta$$

$$[(m_2 - m_1)^{\mathrm{T}} \mathbf{C}_2^{-1} (m_2 - m_1)]^{\frac{1}{2}} < \delta$$



Numerical illustration

Consider nonlinear system with Gaussian noises

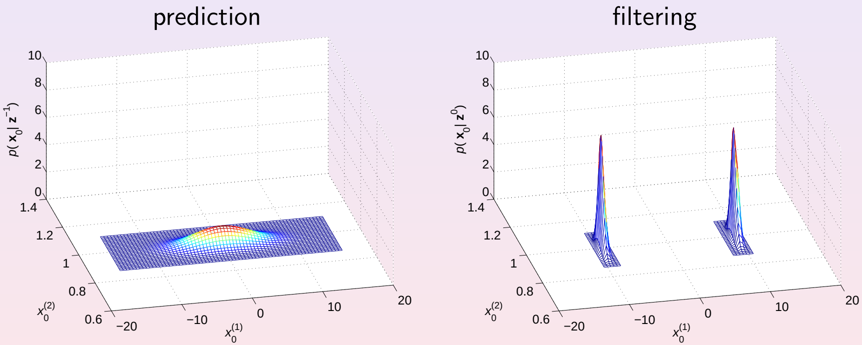
$$
\begin{aligned}
x_{k+1}^{(1)} &= x_k^{(1)} x_k^{(2)} + w_k^{(1)} \\
x_{k+1}^{(2)} &= x_k^{(1)} + w_k^{(2)} \\
z_k &= 0.2\left(x_k^{(2)}\right)^2 + v_k
\end{aligned}
$$

$$
p(w_k) = \mathcal{N}\left\{\mathbf{w}_k; \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0.25 & 0 \\ 0 & 10^{-4} \end{bmatrix}\right\},
$$
$$
p(v_k) = \mathcal{N}\{v_k; 0, 1\},
$$
$$
p(x_0) = \mathcal{N}\left\{\mathbf{x}_0; \begin{bmatrix} 0.1 \\ 0.99 \end{bmatrix}, \begin{bmatrix} 16 & 0 \\ 0 & 0.001 \end{bmatrix}\right\}
$$

$$
p(\mathbf{x}_k | \mathbf{z}^k) = ?
$$

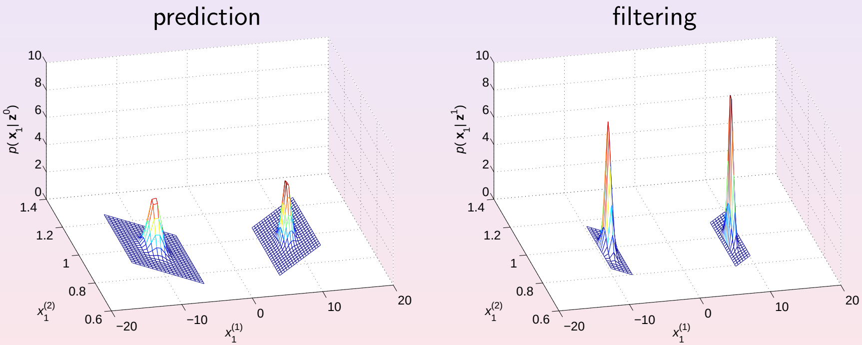
Numerical illustration: Simulation results

$\mathbf{x}_0$



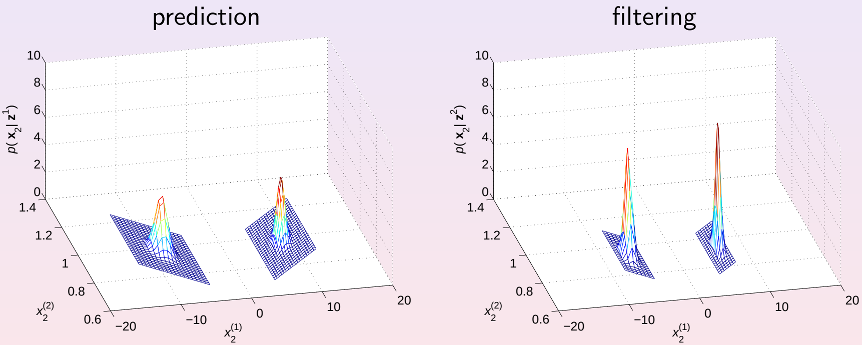
Numerical illustration: Simulation results

$\mathbf{x}_1$



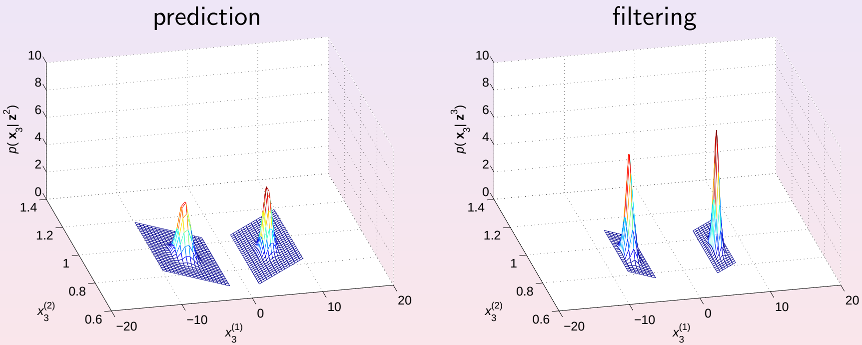
Numerical illustration: Simulation results

$\mathbf{x}_2$



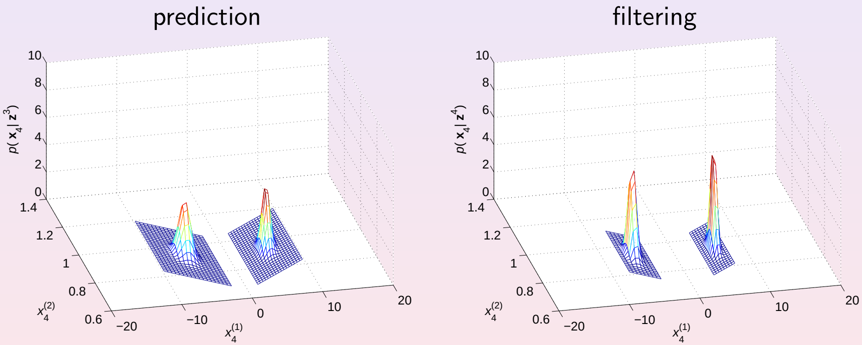
Numerical illustration: Simulation results

$\mathbf{x}_3$



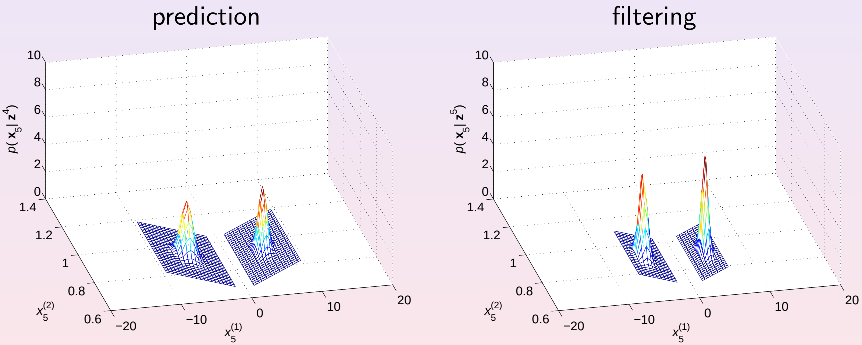
Numerical illustration: Simulation results

$\mathbf{x}_4$

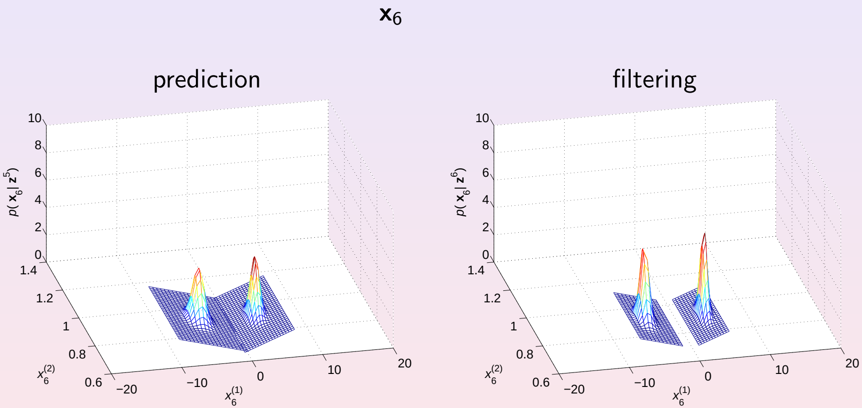


Numerical illustration: Simulation results

$\mathbf{x}_5$

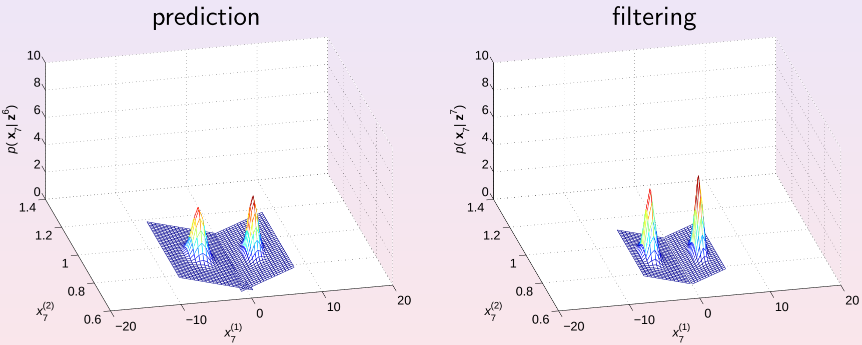


Numerical illustration: Simulation results



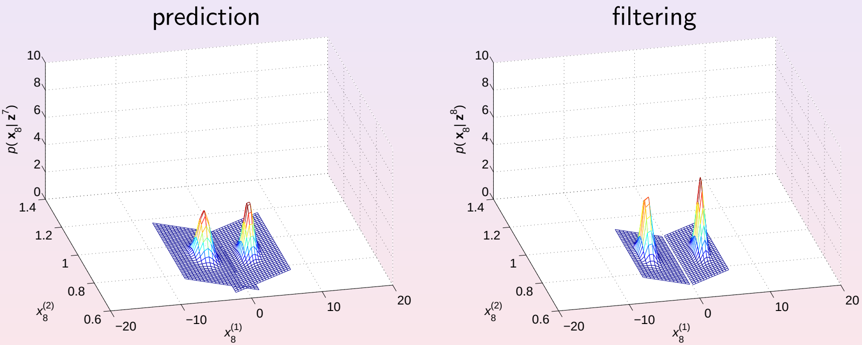
Numerical illustration: Simulation results

$\mathbf{x}_7$



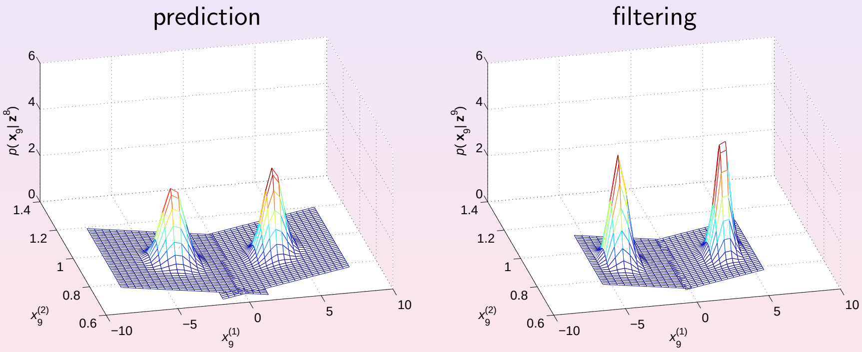
Numerical illustration: Simulation results

$\mathbf{x}_8$



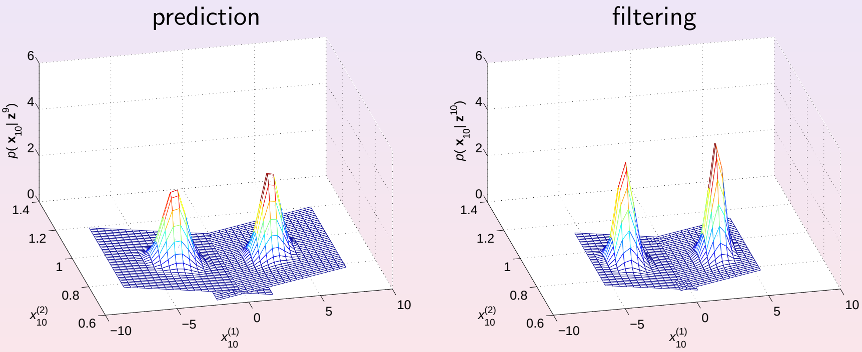
Numerical illustration: Simulation results

$\mathbf{x}_9$



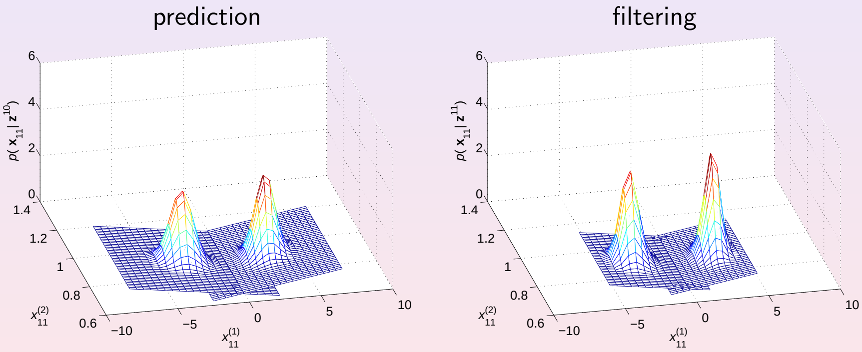
Numerical illustration: Simulation results

$\mathbf{x}_{10}$



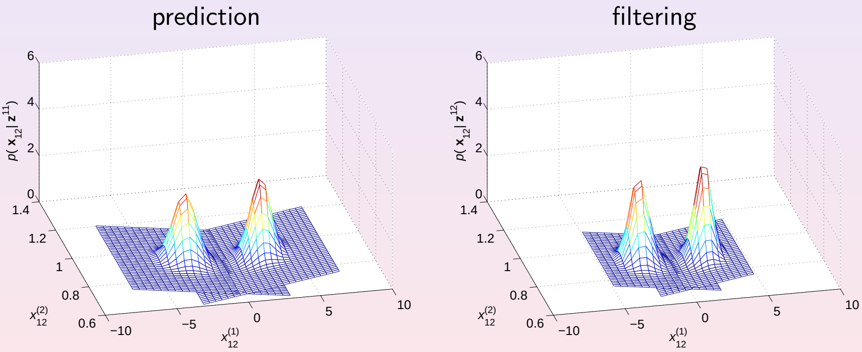
Numerical illustration: Simulation results

$\mathbf{x}_{11}$

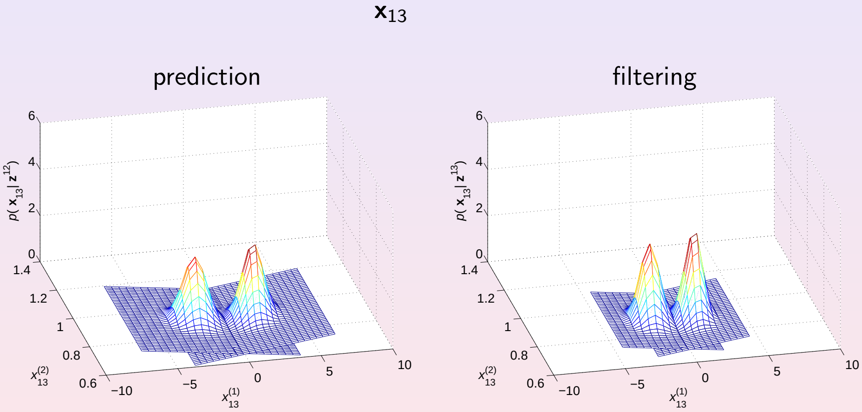


Numerical illustration: Simulation results

$\mathbf{x}_{12}$

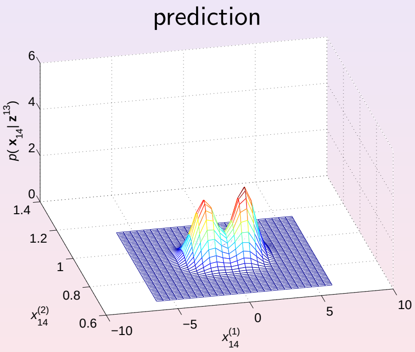


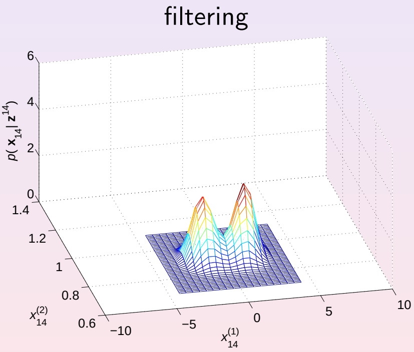
Numerical illustration: Simulation results



Numerical illustration: Simulation results

$\mathbf{x}_{14}$

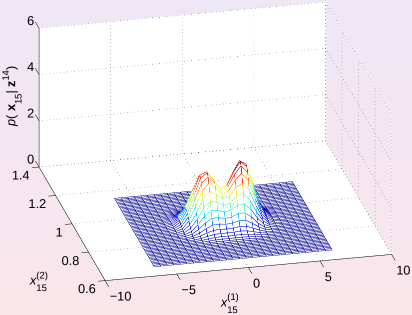




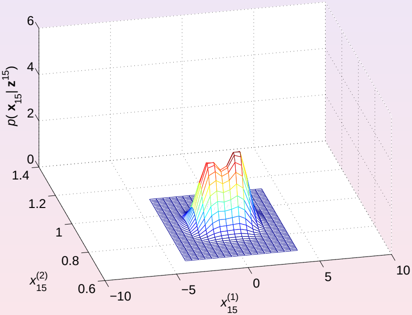
Numerical illustration: Simulation results

$\mathbf{x}_{15}$

prediction



filtering



Numerical illustration: Approximation quality and computational demands

Algorithm	CPU Time (sec)	Avg. V_k	Avg. N_k
Basic	1235	0.0895	4141
Anticipative	182	0.0073	1964
Boundary-Based	*210*	*0.0068*	*1675*
Thrifty Convolution	*59*	*0.0068*	*1675*
Multigrid Design	72	0.0068	702
Particle Filter #1	1	0.7039	500
Particle Filter #2	35	0.4386	4000
Particle Filter #3	6830	0.1105	50000

$$V_k = 1 - \int \min\left\{\hat{p}(\mathbf{x}_k|\mathbf{z}^k), p(\mathbf{x}_k|\mathbf{z}^k)\right\} \mathrm{d}\mathbf{x}_k$$

Conclusion

Advanced point-mass method was presented

- Basic point-mass method
- Anticipative approach
- Multigrid design
- Splitting and merging

1. Šimandl, M., J. Královec and T. Söderström (2002): Anticipative grid design in point-mass approach to nonlinear state estimation. IEEE Transactions on Automatic Control 47(4), 699–702.
2. Šimandl M., Královec J. , Söderström T. (2006): Advanced point mass method for nonlinear state estimation, Automatica 42, Issue 7, 1133–1145