IFAC Symposium "Adaptive Control and Signal Processing", Glasgow 1989

BAYESIAN FILTERING FOR DISCRETE-TIME SYSTEMS WITH RANDOM STRUCTURE

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Abstract. Nonlinear filtering problem for discrete-time systems with random structure is treated by the systematic use of Bayes approach. The main result lies in difference equations for the posterior distribution of the system state and structure index which can serve as a basis for any optimal or suboptimal filter construction. The optimal filtering equations have such a structure that even in the linear Gaussian case optimal finite-dimensional filters do not exist. An approximate filtering problem is considered.

Keywords. Nonlinear filtering, Bayes methods, random structure systems.

INTRODUCTION

Filtering problem for systems with random structure arises in modelling of complex nonlinear industrial processes by a set of linear models each of which is tuned to the specific operation mode of the process. Design of high-performance failure accommodating control systems, estimation with uncertain observations, outlier rejection are examples of the special cases of the general filtering problem, too.

Filtering for continuous-time systems with random structure was studied by Kazakov and Artemjev (1980), Loparo and others (1986). The discrete-time case is considered by Tungait (1982), Blom (1985) and Kliokys (1987). However, Blom (1985), Tungait (1982) and the authors cited in Tungait (1982) concentrated their attention on the restricted versions of the problem and also proposed solutions did not use completely Markov properties of the considered system so as a result the number of conditional filters implementing the optimal filtering algorithm was exponentially growing.

In this paper filtering for discrete-time random structure systems is treated by the systematic use of Bayes approach which has proved to be useful for different estimation (Ho and Lee, 1968; Jazwinski, 1970; Peterka, 1981) and control (Peterka, 1986) problems. The main result of the paper are equations for posterior distribution of the system state and structure index which yield a unified conceptual framework supporting the understanding of diverse special cases and provide the basis for any optimal or suboptimal filter construction. Our solution requires a constant number of conditional filters but the optimal filtering equations have such a structure that even in the linear Gaussian case finite-dimensional conditional filters do not exist. Approximate filtering in the linear Gaussian case is studied. The performance of the approximate filters is evaluated by statistical simulation.

PROBLEM FORMULATION

Consider a discrete-time system with random structure which is described by a hybrid random sequence $\{x_k,\theta_k,z_k,\ k=0,1,\ldots\}$ composed of continuous x_k, z_k and discrete θ_k random variables. Here $x_k\in R^n$ is the state vector, $\theta_k\in\Theta=\{1,2,\ldots,M\}$ is the system structure index and $z_k\in R^m$ is the observation vector. The crucial property of the considered system is expressed by the assumption that the sequence $\{x_k,\theta_k,z_k,k=0,1,\ldots\}$ is Markov. In addition, we suppose that a complete probabilistic description of the random sequence $\{x_k,\theta_k,z_k,k=0,1,\ldots\}$, or equivalently the random structure system, can be defined by the observation conditional probability density functions (cpdf)

$$f(z_k|x_k,\theta_k,\theta_{k-1})\ (\theta_k,\theta_{k-1}\in\Theta), \tag{1}$$

the conditional state transition probability density functions (pdf)

$$q(x_k|x_{k-1},\theta_k,\theta_{k-1})\ (\theta_k,\theta_{k-1}\in\Theta), \tag{2}$$

and the transition probabilities of the Markov chain $\{\theta_k,k=0,1,\ldots\}$

$$\pi(\theta_k|\theta_{k-1})\ (\theta_k,\theta_{k-1}\in\Theta). \tag{3}$$

Also initial conditions must be determined by the observation and state cpdf and by the initial probabilities of the system structure index correspondingly:

$$f(z_o|\theta_o),\ q(x_o|\theta_o),\ \pi(\theta_o)\ (\theta_o\in\Theta). \tag{4}$$

Formulae (1)-(4) determine the model of the system with a random structure and represent the starting point for the filtering problem solution.

The filtering problem is to obtain the optimal estimate of the system state $\{x_k,k=0,1,\ldots\}$ and its structure index $\{\theta_k,k=0,1,\ldots\}$ from the observations $\{z_k,k=0,1,\ldots\}$. Any desired Bayesian estimate of x_k and θ_k can be constructed if conditional a posteriori pdf $w_k(x_k|\theta_k)\equiv f(x_k|\theta_k,Z_o^k)$ and a posteriori probabilities $W_k(\theta_k=j)\equiv P(\theta_k=j|Z_o^k)$ are available. Thus the problem is to find recursive equations for $w_k(x_k|\theta_k)$ and $W_k(\theta_k)$.

EQUATIONS FOR POSTERIOR DISTRIBUTIONS

General Case

Let us introduce the notation for a posteriori pdf of state x_k conditioned on the structure index at two sequential time indices: $w_k(x_k|\theta_k,\theta_{k-1}) \equiv \equiv f(x_k|\theta_k,\theta_{k-1},Z_o^k)$.

Proposition 1. Suppose that the random structure system for $k=0,1,\ldots$ is described by (1)-(4). Then the conditional a posteriori pdf $w_k(x_k|\theta_k)$ and a posteriori probabilities $W_k(\theta_k)$ are defined by the system of functional difference equations:

$$w_k(x_k|\theta_k=j) = \sum_{i=1}^{M} \kappa_k^{(ij)} w_k(x_k|\theta_k=j,\theta_{k-1}=i), \qquad (5)$$

$$w_k(x_k|\theta_k,\theta_{k-1}) = \frac{f(z_k|x_k,\theta_k,\theta_{k-1})\times}{\Psi_{k-1}(z_k|\theta_k,\theta_{k-1})}$$

$$\times \int q(x_k|x_{k-1},\theta_k,\theta_{k-1})\, w_{k-1}(x_{k-1}|\theta_{k-1})dx_{k-1}, \qquad (6)$$

$$W_k(\theta_k=j) = \frac{\sum_{i=1}^{M} \pi(\theta_k=j|\theta_{k-1}=i)\Psi_{k-1}(z_k|\theta_k=j,\theta_{k-1}=i)\times}{\sum_{j=1}^{M}(\text{numerator})}$$

$$\times W_{k-1}(\theta_{k-1}=i)(\theta_k,\theta_{k-1}\in\Theta;\ k=1,2,\ldots), \qquad (7)$$

where

$$\kappa_k^{(ij)} = \frac{\pi(\theta_k=j|\theta_{k-1}=i)\times}{\sum_{i=1}^{M}(\text{numerator})}$$

$$\times \Psi_{k-1}(z_k|\theta_k=j,\theta_{k-1}=i)W_{k-1}(\theta_{k-1}=i), \qquad (8)$$

$$\Psi_{k-1}(z_k|\theta_k,\theta_{k-1}) = \int f(z_k|x_k,\theta_k,\theta_{k-1})\times$$

$$\times \int q(x_k|x_{k-1},\theta_k,\theta_{k-1})w_{k-1}(x_{k-1}|\theta_{k-1})dx_{k-1}dx_k. \qquad (9)$$

The initial conditions are given by $w_o(x_o|\theta_o)$ and $W_o(\theta_o)$.

The proof is based on the Markov property of the sequence $\{x_k,\theta_k,z_k,k=0,1,\ldots\}$, Bayes formula and the basic operations with cpdf, so here is omitted.

The a posteriori cpdf $w_k(x_k|\theta_k)$, $\theta_k\in\Theta$ can be obtained using M^2 parallel branches realizing computations according to (6). Interaction between these branches is defined by (5).

Two special cases of the initial system description (1)-(4) can be distinguished. This is convenient to do because there are situations in practice which can be represented by a more simple description than (1)-(4), and although these cases can be treated by the general model, it is useful to apply a more special initial description which leads to a more simple solution of the filtering problem.

Special Case 1: Observation cpdf Independent of Structure Dynamics

Let us consider the case when observation cpdf are dependent only on the current system structure index, i.e.

$$f(z_k|x_k,\theta_k,\theta_{k-1}) \equiv f(z_k|x_k,\theta_k)\ (\theta_k,\theta_{k-1}\in\Theta) \qquad (10)$$

while the other assumptions (2)-(4) remain unchanged.

Proposition 2. Suppose that the random structure system for $k=0,1,\ldots$ is described by (2)-(4), (10). Then conditional a posteriori pdf $w_k(x_k|\theta_k)$ and a posteriori probabilities $W_k(\theta_k)$ are defined by the system of functional difference equations:

$$w_k(x_k|\theta_k) = \frac{f(z_k|x_k,\theta_k)w_{k-1}(x_k|\theta_k)}{\Psi_{k-1}(z_k|\theta_k)}, \qquad (11)$$

$$w_{k-1}(x_k|\theta_k=j) = \sum_{i=1}^{M} \kappa_k^{(ij)} w_{k-1}(x_k|\theta_k=j,\theta_{k-1}=i), \qquad (12)$$

$$w_{k-1}(x_k|\theta_k,\theta_{k-1}) = \int q(x_k|x_{k-1},\theta_k,\theta_{k-1})\times$$

$$\times w_{k-1}(x_{k-1}|\theta_{k-1})dx_{k-1}, \qquad (13)$$

$$W_k(\theta_k=j) = \frac{\Psi_{k-1}(z_k|\theta_k=j)\sum_{i=1}^{M}\pi(\theta_k=j|\theta_{k-1}=i)\times}{\sum_{j=1}^{M}(\text{numerator})}$$

$$\times W_{k-1}(\theta_{k-1}=i)\ (\theta_k,\theta_{k-1}\in\Theta;\ k=1,2,\ldots), \qquad (14)$$

where

$$\kappa_k^{(ij)} = \frac{\pi(\theta_k=j|\theta_{k-1}=i)W_{k-1}(\theta_{k-1}=i)}{\sum_{i=1}^{M}(\text{numerator})}, \qquad (15)$$

$$\Psi_{k-1}(z_k|\theta_k) = \int f(z_k|x_k,\theta_k)w_{k-1}(x_k|\theta_k)dx_k. \qquad (16)$$

The initial conditions for (13) and (14) are $w_o(x_o|\theta_o)$ and $W_o(\theta_o)$.

In this case we have M^2 equations (13) only for the conditional one-step-ahead prediction pdf $w_{k-1}(x_k|\theta_k,\theta_{k-1})$, while the filtering relations of Proposition 1 define M^2 full-scale conditional filters. The cpdf $w_{k-1}(x_k|\theta_k)$ determined as a weighted sum (12) of cpdf $w_{k-1}(x_k|\theta_k,\theta_{k-1})$ are used in M usual Bayes formulae (11) relating the conditional state prediction $w_{k-1}(x_k|\theta_k)$ and filtering $w_k(x_k|\theta_k)$ cpdf. Thus the structure of filter (11)-(16) is simpler than (5)-(9).

Special Case 2: Observation and State Transition cpdf Independent of Structure Dynamics

The simplest filtering equations are obtained when in addition to (10) the identity

$$q(x_k|x_{k-1},\theta_k,\theta_{k-1}) \equiv q(x_k|x_{k-1},\theta_k)\ (\theta_k,\theta_{k-1}\in\Theta) \qquad (17)$$

is assumed to be valid.

Proposition 3. Suppose that the random structure system for $k=0,1,\ldots$ is described by (3), (4), (10) and (17). Then the conditional a posteriori pdf $w_k(x_k|\theta_k)$ and a posteriori probabilities $W_k(\theta_k)$ are defined by the system of functional difference equations:

$$w_k(x_k|\theta_k) = \frac{f(z_k|x_k,\theta_k)\int q(x_k|x_{k-1},\theta_k)\times}{\Psi_{k-1}(z_k|\theta_k)}$$

$$\times w_{k-1}(x_{k-1}|\theta_k)dx_{k-1}\,, \qquad (18)$$

$$w_{k-1}(x_{k-1}|\theta_k=j)=\sum_{i=1}^{M}\kappa_k^{(ij)}w_{k-1}(x_{k-1}|\theta_{k-1}=i)$$

$$(\theta_k\in\Theta;k=1,2,\ldots), \qquad (19)$$

where

$$\Psi_{k-1}(z_k|\theta_k)=\int f(z_k|x_k,\theta_k)\int q(x_k|x_{k-1},\theta_k)\times$$

$$\times w_{k-1}(x_{k-1}|\theta_k)dx_{k-1}dx_k. \qquad (20)$$

The a posteriori probabilities $W_k(\theta_k)$ and weights $\kappa_k^{(ij)}$ are determined by (14) and (15), respectively. The initial conditions are $w_o(x_o|\theta_o)$ and $W_o(\theta_o)$.

The computation of the conditional a posteriori pdf $w_k(x_k|\theta_k)$ can be realised by the bank of M interacting conditional filters, defined by (18).

Discussion

The common feature of all three filtering relations, determined by Propositions 1-3, is the presence of the weighted sum operation on cpdf in the recursive equations. This means that no finite-dimensional optimal filter solving the above problem exists for any cpdf (1), (2) and any distribution of initial conditions (4). Regardless of this, the results of the Propositions 1-3 are a convenient starting point for the design of approximate finite-dimensional filters for diverse nonlinear filtering problems.

APPROXIMATE FILTERING IN THE LINEAR GAUSSIAN CASE

Because of the limited volume of the paper only the case of Proposition 1 will be considered here. Similar results can be easily obtained for Propositions 2, 3.

Let us assume that the cpdf (1), (2), (4) are Gaussian. A simple approximate filter can be obtained if cpdf $w_k(x_k|\theta_k)$ are approximated by Gaussian densities. We shall use two ways to maintain Gaussian form of these cpdf during the filtering recursion.

Gaussian Approximation

Consider the approximate filtering problem for Proposition 1. From (5) it can be easily seen that the conditional (with respect to $\theta_k=j$) a posteriori means $\hat{x}_{k|k}^{(j)}$ and covariance matrices $P_{k|k}^{(j)}$ can be expressed through the conditional (with respect to $\theta_k=j$, $\theta_{k-1}=i$) a posteriori means $\hat{x}_{k|k}^{(ij)}$ and covariance matrices $P_{k|k}^{(ij)}$ according to

$$\hat{x}_{k|k}^{(j)}=\sum_{i=1}^{M}\kappa_k^{(ij)}\,\hat{x}_{k|k}^{(ij)}\,, \qquad (21)$$

$$P_{k|k}^{(j)}=\sum_{i=1}^{M}\kappa_k^{(ij)}\{P_{k|k}^{(ij)}+\hat{x}_{k|k}^{(ij)}\,\hat{x}_{k|k}^{(ij)T}\}-$$

$$-\hat{x}_{k|k}^{(j)}\,\hat{x}_{k|k}^{(j)T}\,. \qquad (22)$$

Then approximation of $w_k(x_k|\theta_k)$ for each time index k by Gaussian densities with parameters $\hat{x}_{k|k}^{(j)}$, $P_{k|k}^{(j)}$, defined by (21) and (22), and substitution of Gaussian pdf (1)-(2) into (5)-(9) yield the following approximate filter:

1. The conditional (with respect to $\theta_k=j$, $\theta_{k-1}=i$) estimates $\hat{x}_{k|k}^{(ij)}$ and associated matrices $P_{k|k}^{(ij)}$ are determined by the Kalman filter equations

$$[x_{k|k}^{(ij)},P_{k|k}^{(ij)}]=KF^{(ij)}(\hat{x}_{k-1|k-1}^{(i)},P_{k-1|k-1}^{(i)},z_k)$$

$$(i,j\in\Theta;\ k=1,2,\ldots), \qquad (23)$$

where $KF^{(ij)}(\cdot)$ is the Kalman filter operator performing time and measurement updating of $\hat{x}_{k-1|k-1}^{(i)}$, $P_{k-1|k-1}^{(i)}$ on the basis of model (1) and (2). Substitution of Gaussian densities into (9) yields the Gaussian density with a zero mean and covariance matrix $V_k^{(ij)}$

$$\Psi_{k-1}(z_k|\theta_k=j,\ \theta_{k-1}=i)=N[\nu_k^{(ij)}|0,V_k^{(ij)}], \qquad (24)$$

where $\nu_k^{(ij)}$ and $V_k^{(ij)}$ are the measurement residual and its covariance matrix produced by the corresponding conditional Kalman filter;

2. The conditional (with respect to $\theta_k=j$) estimates $\hat{x}_{k|k}^{(j)}$ and assotiated matrices $P_{k|k}^{(j)}$ are expressed by the weighted sums (21) and (22) with the weights (8) and with (24) faken into account;

3. The a posteriori probabilities $W_k(\theta_k)$ of the structure index are given by the system of recursive equations (7) with (24) taken into account;

4. The unconditional estimate $\hat{x}_{k|k}$ is determined

$$\hat{x}_{k|k}=\sum_{j=1}^{M}W_k(\theta_k=j)\,\hat{x}_{k|k}^{(j)}\,. \qquad (25)$$

<u>Remark.</u> Although in the description of the algorithm the terms "mean value" and "covariance matrix" are used actually we have here only approximation of the true characteristics.

Multiplicative Gaussian Mixture Approximation

Now we shall introduce some auxiliary results from the statistical decision theory which provide motivation for the approximate filter construction. Consider Bayesian methodology (DeGroot, 1970) for the statistical decision problem which consists of the choice of a single pdf f(x) from a given finite set $\Omega=\{f_1(x),\ldots,f_M(x)\}$ with probability distribution $p=\{p_1,\ldots,p_M\}$ defined on Ω. Let us consider two loss functionals which are the logarithmic divergences of two probability distributions (Cziszár, 1967):

$$L_1[f(x),f_i(x)]=\int f_i(x)\ln\frac{f_i(x)}{f(x)}dx \qquad (26)$$

and

$$L_2[f_i(x),f(x)]=\int f(x)\ln\frac{f(x)}{f_i(x)}dx\,. \qquad (27)$$

The risks corresponding to the above presented loss functionals are defined by

$$\rho_j[p, f(x)] = \sum_{i=1}^{M} p_i L_j[f(x), f_i(x)] \quad (j=1,2). \qquad (28)$$

Lemma. If the product of the pdf $f_i(x) \in \Omega$ for all i such that $\rho_j > 0$ is not zero a.e., then the unique minimum of the risks (28) can be achieved and the minimizing pdf are:

- in the case of loss functional (26)

$$f^*(x) = \sum_{i=1}^{M} p_i f_i(x) ; \qquad (29)$$

- in the case of loss functional (27)

$$f^*(x) \propto \prod_{i=1}^{M} [f_i(x)]^{p_i} , \qquad (30)$$

where $\propto$ means equality up to a normalizing constant independent of x .

The proof of the Lemma can be done by direct substitution of pdf (29) and (30) into (28) and utilizing the basic properties of divergences (see Csiszár, 1967; cf. Kulhavý, 1987).

Let us return now to the filtering problem. Relation (5) determining the cpdf $w_k(x_k|\theta_k=j)$, $j=\overline{1,M}$ can be interpreted as the solution for each time index k of the above discussed statistical decision problem for the loss functional (26) with

$\Omega = \{w_k(x_k|\theta_k=j, \theta_{k-1}=1), \ldots, w_k(x_k|\theta_k=j, \theta_{k-1}=M)\}$

and $p = \{\kappa_k^{(1j)}, \ldots, \kappa_k^{(Mj)}\}$ (cf (5) and (29). It is easy to see from (8) that the weights $\kappa_k^{(ij)}$ in (5) are conditional probabilities, i.e. $\kappa_k^{(ij)} = P(\theta_{k-1}=i|\theta_k=j, Z_o^k)$. A similar interpretation is valid for formulae (12) and (19). The additive mixture operations (5), (12) and (19) destroy the initial Gaussian densities during recursion and are the cause of difficulties for a finite-dimensional filter construction. It is desirable to replace these operations by other ones which retain Gaussian cpdf. The motivation for this replacement is the above presented Lemma. Indeed, if in the optimal filtering relations the solution of the decision problem is based on the loss functional (26), then we can try to solve the same decision problem but with the loss functional defined by (27). The solution of such a modified decision problem leads to the multiplicative mixture of densities (30) and in the linear Gaussian case the recursive relations of Propositions 1-3 reproduce the initial Gaussian conditional pdf $q(x_o|\theta_o)$ without the need for any other approximation. Then the only change in the filtering equations of the Proposition 1 consists in the replacement of (1) by

$$w_k(x_k|\theta_k=j) \propto \prod_{i=1}^{M} [w_k(x_k|\theta_k=j, \theta_{k-1}=i)]^{\kappa_k^{(ij)}} . \qquad (31)$$

Now, taking into assount (31) the approximate filter based on Proposition 1 can be defined by steps 1-4 of the previous section but with the replacement of (21) and (22) by

$$(P_{k|k}^{(j)})^{-1} \hat{x}_{k|k}^{(j)} = \sum_{i=1}^{M} \kappa_k^{(ij)} (P_{k|k}^{(ij)})^{-1} \hat{x}_{k|k}^{(ij)} , \qquad (32)$$

$$(P_{k|k}^{(j)})^{-1} = \sum_{i=1}^{M} \kappa_k^{(ij)} (P_{k|k}^{(ij)})^{-1} . \qquad (33)$$

SIMULATION EXAMPLES

Two simulation examples are considered. They provide a comparison between the performance of the optimal Kalman filters based on the observed sequence $\{\theta_k, k=0,1,\ldots\}$, the approximate filters, based on the Gaussian approximation of a posteriori pdf (F1 - filters) and the filters, obtained using the multiplicative Gaussian mixture approximation of cpdf (F2 - filters). Let us denote a squared estimation errors averaged over 5000 samples of time history by σ_o^2 (for the optimal Kalman filter), σ_1^2 (for F1 -filter) and σ_2^2 (for F2 - filter). Then as performance indicators the relative values $\rho_i = \sigma_i^2/\sigma_o^2$, $i=1,2$ are used which can be compared with the ideal case $\rho_o=1$.

Example 1. Consider the filtering problem for the scalar process with abruptly changing correlation characteristics but a constant energy. The process and observation models are

$$x_{k+1} = \Phi^{(j)} x_k + \xi_k , \quad z_k = x_k + \eta_k \quad (j=1,2)$$

where $\{\xi_k\}$, $\{\eta_k\}$ are uncorrelated Gaussian sequences with variances Q and R . The transition probability matrix is symmetric with the diagonal elements equal to 0.9. Variance Q was chosen to ensure a unit steady - state variance of $\{x_k\}$. Performance indicators ρ_i, $i=1,2$ are presented for different signal-to-noise rations and different parameters $\Phi^{(j)}$ in Table 1.

Example 2. This example illustrates application of the above presented algorithms to the filtering problem with measurements containing outliers. The simulated processes are given by equations

$$x_{k+1} = 0.9\, x_k + \xi_k , \quad z_k = x_k + \eta_k^{(j)} \quad (j=1,2) ,$$

where $\{\xi_k\}$, $\{\eta_k^{(1)}\}$ and $\{\eta_k^{(2)}\}$ are Gaussian sequences with the variances Q , $R^{(1)} = 1$ and $R^{(2)} \gg R^{(1)}$. The sequence $\{\theta_k\}$ is composed of independent random variables with the probability of outlier $P(\theta_k=2)=0.1$. The performance of the filters is compared (Table 2) for different signal-to-noise rations and different outlier intensities.

SUMMARY AND CONCLUSIONS

The unified treatment of the nonlinear filtering problem for discrete-time systems with random structure has been performed by the use of Bayes approach. The presented solution have such a structure that the optimal filtering algorithm can be constructed on the basis of a finite number of interacting conditional filters operating in parallel. However, even in the linear Gaussian case, optimal finite-dimensional conditional filters do not exist. Two approaches to the design of simple and practically applicable approximate filters have been studied.

Simulation examples have been presented comparing different filters. The results indicate that the performance of F1 and F2 - filters is very much dependent upon the system model and simulation conditions. The accuracy of F1 and F2 - filters is comparable and there are situations when F1 - filter gives better results than F2 - filter and vice versa. Therefore, no single algorithm can be recommended for all situations from the accuracy point of view. As concerns the computational effort, F1 - filters are simpler for system models in which the dimension of the state vector is greater than the dimension of the observation vector and the covariance form of the Kalman filter is more preferable. On the other hand, in the case of measurement redundancy, when the information form of the Kalman filter is more preferable, the implementation of F2-filters is simple.

Finally, considering application possibilities, the results presented in this paper were used in design of the algorithm for estimating abruptly changing parameters of a regressiontype models (Kaminskiene and Kliokys, 1987). Applications to the power system transformer tap position tracking and to state estimation of ARMA/Delta models with jump-parameters are in progress.

ACKNOWLEDGMENTS

The first author's visit in Czechoslovakia has been made possible through financial support by the Czechoslovak Academy of Sciences. The authors also wish to acknowledge V.Peterka for helpful discussions.

REFERENCES

Blom, H.A.P. (1985). An efficient decision-making-free filter for processes with abrupt changes. Prep. 7th IFAC/IFORS Symp. on Identification and System Parameter Estimation, York, pp. 631-636.

Cziszár, I. (1967). Information-type measures of difference of probability distributions and indirect observations. Studia Sci. Math. Hungarica, 2, 299-318.

DeGroot, M.H. (1970). Optimal Statistical Decisions. McGraw-Hill, New York.

Ho, Y.C. and R.C.K. Lee (1964). A Bayesian approach to problems in stochastic estimation and control. IEEE Trans. Aut. Control, 9, 333-339.

Jazwinski, A.H. (1970). Stochastic Process and Filtering Theory. Academic Press, New York.

Kaminskiene, J. and E. Kliokys (1987). Identification of linear dynamic systems and processes with parameters subjected to abrupt changes. Proc. of Lithuanian Academy of Sciences. Series B, 5(162), 102-113 (in Russian).

Kazakov, I.E. and V.M. Artemjev (1980). Optimization of Dynamic Systems with Random Structure. Nauka, Moscow (in Russian).

Kliokys, E. (1987). Optimal filtering in discrete-time random structure systems. Automation and Remote Control, 11, 61-70 (in Russian).

Kulhavý, R. (1987). Restricted exponential forgetting in real-time identification. Automatica, 23, 589-600.

Loparo, K., Z. Roth and S. Eckert (1986). Nonlinear filtering for systems with random structure. IEEE Trans. Aut. Control, 31, 1064-1068.

Peterka, V. (1981). Bayesian approach to system identification. In P. Eykhoff (Ed.), Trends and Progress in System Identification. Pergamon Press, Oxford, Chap. 8, pp. 239-304.

Peterka, V. (1986). Control of uncertain processes: applied theory and algorithms. Supplement to the journal Kybernetika, 22, no.3-6.

Tungait, J.K. (1982). Detection and estimation for abruptly changing systems. Automatica, 18, 607-615.

TABLE 1 Performance Indicators for Jump-parameter Process Model

$\phi^{(1)}$	$\phi^{(2)}$	R	ρ_1	ρ_2
0.9	0.1	0.05	1.0400	1.0389
		0.1	1.0643	1.0624
		0.5	1.1239	1.1318
0.9	0.8	0.05	1.0138	1.0134
		0.1	1.0198	1.0193
		0.5	1.0228	1.0218

TABLE 2 Performance Indicators for the Case of Measurement Containing Outliers

Q	$R^{(2)}$	ρ_1	ρ_2
0.019	5	1.0367	1.0387
	10	1.0441	1.0508
	50	1.0488	1.0555
0.19	5	1.0651	1.0670
	10	1.1053	1.1066
	50	1.1365	1.1392
1.9	5	1.1288	1.1484
	10	1.2477	1.2526
	50	1.4567	1.4199