IFAC Workshop "Evaluation of Adaptive Control Strategies in Industrial Applications", Tbilisi 1989

PRACTICALLY-ORIENTED LQ SELF-TUNERS

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Abstract. The LQG theory of adaptive control is now well developed. However, to be applicable in real system control, specific measures usually have to be taken. Based on simulation experience and several practical applications, the authors have designed a self-tuning controller that respects most of practice-oriented requirements.

Keywords. Adaptive control; self-tuning regulators; recursive estimation; Bayes methods.

INTRODUCTION

The LQG controllers based on the certainty equivalence principle represent a broad stream in designing self-tuning controllers.

In fact, the LQG approach is now well theoretically developed and its properties have been analysed in detail. However, a few practical applications based on this approach have been reported.

Serious objections against the LQG self-tuning control raised by practicians are its theoretical and computational complexity, lack of knowledge how to tailor general algorithms to a specific problem and in the last but not least lack of operational experience. On the other hand, the possibility to reach a better behaviour of the controlled process even under its uncomplete knowledge is very attractive. Hence, the task addressed in the paper, i.e.

> design and modifications of the general self-tuning LQG controller that would meet practicians' requirements,

is very challenging.

A couple of problems have to be solved to obtain a control algorithm that can be widely used in practice. They can be divided to the following groups.

1. *Reliability* is a dominant requirement for the intended use. At present, the hardware support is sufficient in this respect. Thus, the adequacy of the control problem formulation and of the numerical implementation are decisive.
2. *Completeness* of the overall design which has to cover various subproblems (usually taken as extensions of the basic LQG solution) is needed to satisfy specific user's problems.

   The reported algorithm is believed to meet a substantial portion of both the requirements.
3. *Simplicity* of the use is probably the most important prerequisite for the intended controller.

   It has forced us to restrict the number of user's knobs as much as possible.

BASIC PROPERTIES OF CONTROLLER

In our prototype, we rely on consistency of the Bayes-based decision theory and long-lasted positive experience with its application to the flexible linear normal regression model and quadratic multistep criteria.

The majority of problems related to the implementation of such self-tuning controllers is now theoretically resolved. In our design, we employ the below listed achievements.

Reliability is supported by the use of:

- a methodology for incorporating user's prior information about global characteristic of the controlled process (Kárný, 1984),
- a simple regression local filter which removes measurement noise adding negligible dynamics into the closed loop (Hebký, 1984),
- a numerically safe factorized form of recursive parameter estimation which admits reliable parameter tracking; the use of restricted forgetting avoids blowing-up and radically reduces the danger of bursting phenomena known in connection with the use of exponential forgetting (Kulhavý, 1987),
- a method of utilizing prior information about parameter variations in parameter tracking (Kulhavý, 1986),
- a numerically very robust form of multistep quadratic optimization, formally equivalent to a factorized solution of the Riccati equation (Kárný and others, 1985).

The practice-oriented extensions comprise:

- an efficient way of respecting hard restrictions on inputs and/or its changes, which, at the same time, reduces computational demands of the multistep optimization (Böhm and Kárný, 1982; Böhm, 1985),
- an identification of the absolute term in regression model,
- a consideration of nonzero set-point signal with a possibility of the controller pre-programming,
- an offset compensation,
- a consideration of external measurable disturbances (feed-forward signal).

The chosen configuration lets the user to adjust the following performance-oriented knobs:

- the model order,
- the expected rate of static gain changes (relative to other parameters),
- the control period (a multiple of the sampling period),
- the time delay (a multiple of the control period),
- the range of admissible input changes.

Other variables (initial conditions, forgetting factor, control strategy, control horizon etc.) are fixed at widely acceptable values.

BRIEF OVERWIEW OF UNDERLYING THEORY

The extensions of the basic LQG formulation demand the model of the controlled system in the form

$$y_t = \sum_{i=1}^{m_r} a_i y_{t-i} + \sum_{i=0}^{m_r} b_i u_{t-i} + \sum_{i=1}^{l_v} d_i v_{t-i} + k + e_t \qquad (1)$$

where y_t, u_t, v_t, e_t stand for the system output, input, measurable disturbance and unmeasurable white noise, respectively. $\{a_i, b_i, d_i\}$ are the corresponding coefficients of the regression model, m_r denotes the model order, l_v specifies the number of delayed external disturbances assumed, k represents a possible d.c. level in the system. The equation (1) can be written in the short-cut form

$$y_t = P' z_t + e_t = b_0 u_t + P'_x x_{t-1} + P'_v x_{v,t-1} + k + e_t \qquad (2)$$

where

$$\begin{aligned} P' &= [b_0, a_1, b_1, ..., a_{m_r}, b_{m_r}, d_1, ..., d_{l_v}, k], \\ z_t &= [u_t, x'_{t-1}]', \\ x'_{t-1} &= [y_{t-1}, u_{t-1}, ..., y_{t-m_r}, u_{t-m_r}], \\ x'_{v,t-1} &= [v_{t-1}, ..., v_{t-l_v}], \\ P'_x &= [a_1, b_1, ..., a_{m_r}, b_{m_r}], \\ P'_v &= [d_1, ..., d_{l_v}]. \end{aligned}$$

The synthesis of the control law is based on minimizing the multistep quadratic criterion

$$\mathrm{E}\left\{ \sum_{t=t_0+1}^{t_0+N} \left[q_y (y_t - w_t)^2 + q_u (u_t - u_0)^2 \right] + \sum_{t=t_0+N-m_r}^{t_0+N} q_s (u_t - u_0)^2 \right\} \qquad (3)$$

where

E denotes expectation,

t_0, N are initial time and control design horizon, respectively,

$q_y, q_u \geq 0$ and $q_s > 0$ are weights of corresponding deviations,

w_t is a set-point value,

u_0 is an input reference value.

Minimization of the criterion (3) for known parameters of the model (2) leads to a version of the well-known Riccati equation, containing not only quadratic but also linear and absolute terms (Böhm, 1988).

In the resulting control law L, parts depending on $x_t, x_{v,t}, k, w_t$ and u_0 can be distinguished. The optimal u_t is generated by

$$u_t = -L'_x x_{t-1} - L'_v x_{v,t-1} - L_k k - L_w w_t - L_{u_0} u_0. \qquad (4)$$

Parameter estimation is performed using a recursive least-squares algorithm modified by so-called restricted forgetting (Kulhavý, 1987)

$$\hat{P}_t = \hat{P}_{t-1} + \frac{C_{t-1} z_t}{1 + z'_t C_{t-1} z_t} \left(y_t - \hat{P}'_{t-1} z_t \right), \qquad (5)$$

$$C_t^{-1} = C_{t-1}^{-1} + \gamma_t z_t z'_t. \qquad (6)$$

where $\hat{P}_t$ denotes the least-squares estimate and C_t its covariance. The scalar weight γ_t is defined as

$$\gamma_t = \varphi - (1 - \varphi) \left[(z'_t C_{t-1} z_t)^{-1} - (z'_t C^*_{t-1} z_t)^{-1} \right]. \qquad (7)$$

Here φ stands for a forgetting factor ($0 < \varphi \leq 1$). C^*_t represents a "reference" (normalized) covariance matrix of unknown parameters P_t that is introduced to employ prior information about the time evolution of P_t (Kulhavý, 1986).

To incorporate information about static gain variations, we specified C^*_t as follows

$$C^*_t = \alpha \left[I - (1 - \beta) \frac{\tilde{z}_t \tilde{z}'_t}{\tilde{z}'_t \tilde{z}_t} \right] \qquad (8)$$

— the scalar $\alpha > 0$ is fixed to a sufficiently large value,
— the fictitious regressor $\tilde{z}_t$ is defined by replacing all inputs with 1, external disturbances by zeroes and outputs by the posterior estimate of the static gain (computed using the entries of $\hat{P}_t$),
— the scalar $\beta \in [0, 1]$ is a user's knob.

Note that the choice $\beta = 1$ admits free changes of the static gain estimate. The opposite extreme, $\beta = 0$, freezes the static gain estimate to its prior value.

Remarks

1. It is known that some authors prefer the incremental version of the regression model (1) which has the following advantages

   - identification of the coefficient k is avoided,
   - no offset occurs for a set-point change having step form.

   However, simulations indicate that the incremental model is less robust with respect to unmodelled dynamics and more sensitive to the choice of the sampling rate.

2. The acceptable computational burden restricts the horizon to a finite value. However, the multistep criterion does not guarantee stability for a finite N. In the proposed algorithm, the stability is ensured by the following measures.

   - A large penalization of terminal inputs q_s is chosen to guarantee stability for $N > m_r$ (Poubelle and others, 1988).
   - The algorithm is organized so that even with a finite N, usually $N = 1$, the control law tends to the control law corresponding to the infinite-horizon criterion. This strategy is called "iterations spread in time" (Kárný and others, 1985). Its basic idea is to use the final result from minimization as an initial value for the next step.

3. It is shown in Böhm (1988) that u_0 can be simply choosen (as a result of additional minimization of the criterion (3) by u_0) so that it compensates possible offset. A declared disadvantage of the positional (non-incremental) model is removed in this way.

4. Time delay in the system may be treated in two different ways. The first one can be deduced directly from the regression model form. If the system has some time delay,

the corresponding number of b_i parameters is equal to zero. The only necessary assumption is to choose a sufficiently high order of the model to cover the delay. In such a case we do not need to know the time delay precisely. The disadvantage is the substantial increase of computational burden.

The second possibility is to minimize the criterion (3) that admits a noncausal control law. Unavailable data are replaced with their predictions calculated using (1). This approach requires a priori knowledge of the delay, but is much simple computationally. Obviously, both the above possibilities can be combined where suitable.

5. Input saturation is the most frequent type of nonlinearity and the LQG design cannot handle it directly. In our case nonlinear input restriction is recalculated to an additional penalization of the input so that the recalculated input satisfies the restriction (Böhm and Kárný, 1982; Böhm, 1988). By such a data-dependent penalization the nonlinear problem is transformed into linear one.

ALGORITHMS

A theoretically correct design cannot be implemented successfully in practice unless it is realized by algorithms that are both efficient and numerically robust. The proposed algorithms achieve this by using matrix-factorization methods (Bierman, 1977; Kárný and others, 1985) both for identification (least squares) and control synthesis (Riccati equation solution).

CHOICE OF ADJUSTABLE PARAMETERS

There is a lot of parameters that influence the behaviour of the self-tuning control algorithm especially at its start-up. We select a rather limited set of parameters the choice of which is straightforward for the user. The remaining parameters are fixed at values which should fit to the majority of practically met cases. They can be tuned as well, but they need deeper knowledge of the system as well as of the controller (e.g. a priori knowledge of the system parameters). The adjustable parameters follow.

Model order m_r

A higher order model gives the better possibility of a more precise modelling of the process (the system and the disturbance) and makes it possible to obtain better control. The presented algorithms are not sensitive to over-parametrization.

On the other hand, the order of the model reflects the complexity of the model and, consequently, also the computational burden of identification and control synthesis. Thus, the limiting factor for the choice of model order is the available computational time within single sampling and/or control period.

Relative rate of static gain variations

The initial conditions for parameter estimation are fixed in the algorithm. The only knob that is free to influence the resulting estimation is the scalar β in (8) which specifies the relative rate of static gain variations compared to other model parameters. The damping of the static gain estimate has turned out to be a successful measure to moderate, to a certain extent, troubles caused by mismodelling. Thus, tending with β to 0, it is possible to extend the range of satisfactory operation of the controller. Typically, a lower order or faster sampling of the controlled plant is allowed. The rule of thumb is: use $\beta = 1$ first and in case of troubles, when other possibilities are exhausted, decrease β towards 0.

Control period T_s

This parameter relates frequency of the output sampling to the frequency of generating the input. The lowest value is 1, indicating that after each output measurement new input is calculated. If $T_s > 1$, T_s successive output values are used to calculate filtered output $\tilde{y}_t$ that is used further in identification and control.

The higher T_s is the more are high frequency components filtered out from the output. Of course, the control period T_s must be related properly to the dynamics of the system. Either representative data records are available and the proper control period can be estimated (Kárný, 1990) or some experince and experimentations are needed for its choice.

Time delay

It has been said (see Remark 4) that the controller may process the time delay in two ways. When explicitly choosing $T_d > 0$, unknown data are automatically predicted.

Penalization of input q_u

It represents the penalization of the input itself, if u_0 is constant, or of its increment, if $u_0 = u_{t-1}$. In both cases, a higher value of q_u leads to lower values of input or its increments. This value serves as a "real-time" knob by which the user tunes the closed-loop behaviour according to his subjective desire during the normal operation of the controller.

Input signal constraints

The hardware limits are usually known. It is sometimes reasonable to set up even more severe values which result in a more quiet input signal acceptable from technological or economical considerations.

OPERATION MODES OF CONTROLLER

The action of the controller comprises four tasks

- initialization
- data acquisition
- identification
- self-tuning control

Any practical controller cannot start its operation with completely unknown parameters of the model. Either fairly good a priori knowledge must be incorporated before the start of self-tuning control or some preliminary identification has to be done.

Initialization. This mode sets up all default or user-defined initial conditions, it prepares all needed data arrays and puts zeroes where necessary.

Data acquisition. During this mode, actual measured data are fed into the appropriate vectors. Input signal is generated by another controller, or is determined by a suitable random-signal generator, or is man-operated.

Identification. This mode differs from the previous one only by activating identification. The system remains in open loop (when stable) or the loop is closed by another controller. Achieving good parameter estimates requires a sufficiently exciting input signal.

Self-tuning control. In this last mode, the optimal input calculation is started and the self-tuning control loop is closed. If a rather quiet transition to the closed loop behaviour is desired, a very high input penalization is to be choosen for several steps.

If the controller is restarted, the data acquisition can be followed directly by self-tuning control.

IMPLEMENTATION AND EXPERIMENTS

The program for the presented controller is composed of subroutines solving four main tasks. The organization of computation can be followed from the flowchart shown in Fig. 1. The algorithm is available now in FORTRAN and ASSEMBLER 80. Executable module for the microprocessor 8080 occupies about 5kB of memory. The computational burden depends mainly on the selected model order and can be estimated roughly by 0.2 sec for the second order model.

Properties of this controller have been tested intensively by simulations. From a lot of experiments demonstrating different features of the controller we select the comparison of the behaviour of the self-tuner and PID controller.

In both examples a continuous system having the transfer function

$$S(p) = \frac{1}{(p+1)^3}$$

was controlled using the sampling rate of 0.01 sec. The control rate was the same ($T_s = 1$). In Fig. 2 the behaviour of the self-tuner is demonstrated. We chose the model order $m_r = 3$, a very low penalization $q_u = 0.0001$ and input constraints (-20,20). In the first graph the controlled output, set-point value and the disturbance (equal to uncontrolled output) can be seen. The second graph shows the input signal. Note the open-loop action with white noise excitation of low amplitude over the first 50 samples.

Fig. 3 shows the same system controlled by the PID controller which was adjusted by trials. Parameters of this PID are $P = 4.$, $I = .1$, $D = 100$ and $T_d = 0.1$.

CONCLUSION

An attempt has been made to modify standard LQG approach so that it can better meet specific conditions of real situations. In spite of adaptivity features, a careful choice of adjustable parameters is necessary. The crucial parameters are the model order and the sampling period. Both of them can be a priori estimated if representative data records are available (Kárný, 1983; Kárný, 1990). Otherwise some experience and trials are necessary.

This controller has been used in several full-scale experiments with real plants and in some cases it was left in permanent operation. Two of such cases are described in (Švarc, 1989; Fessl, 1985).

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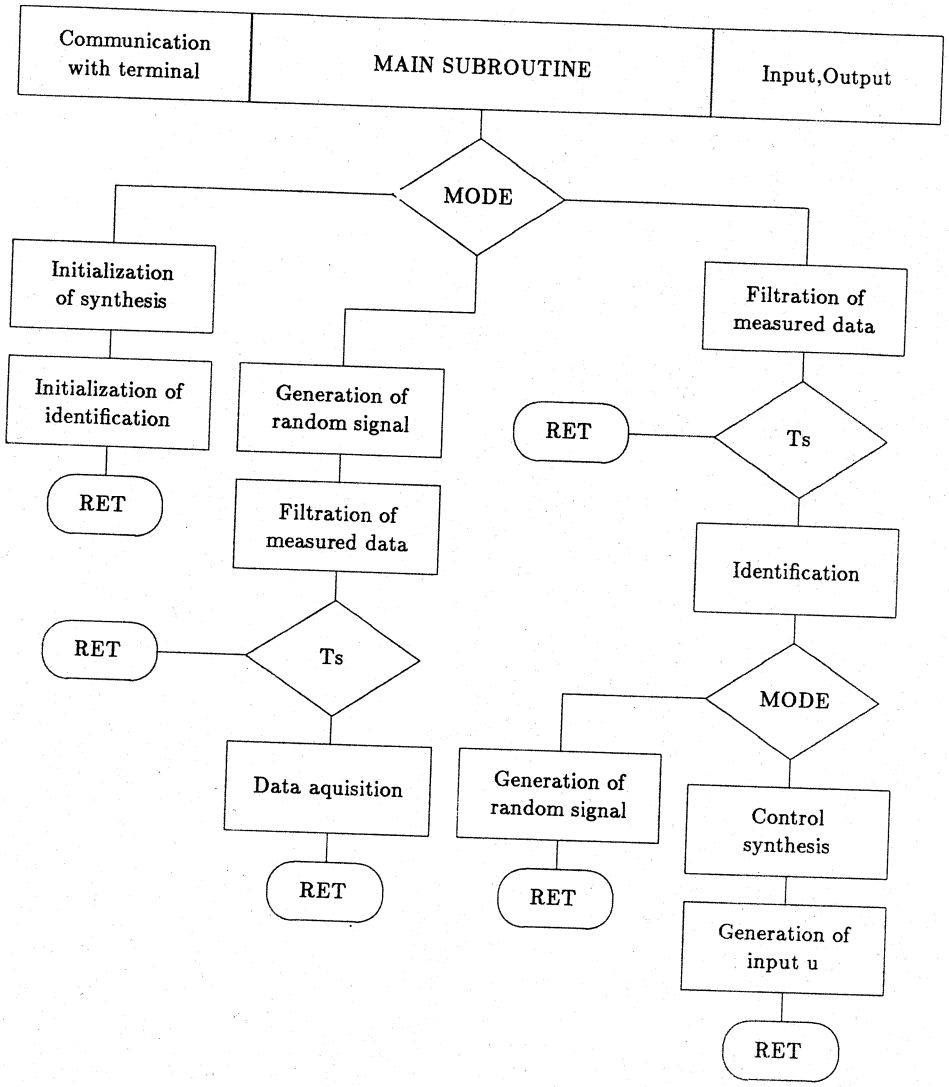

Fig. 1. The controller flowchart

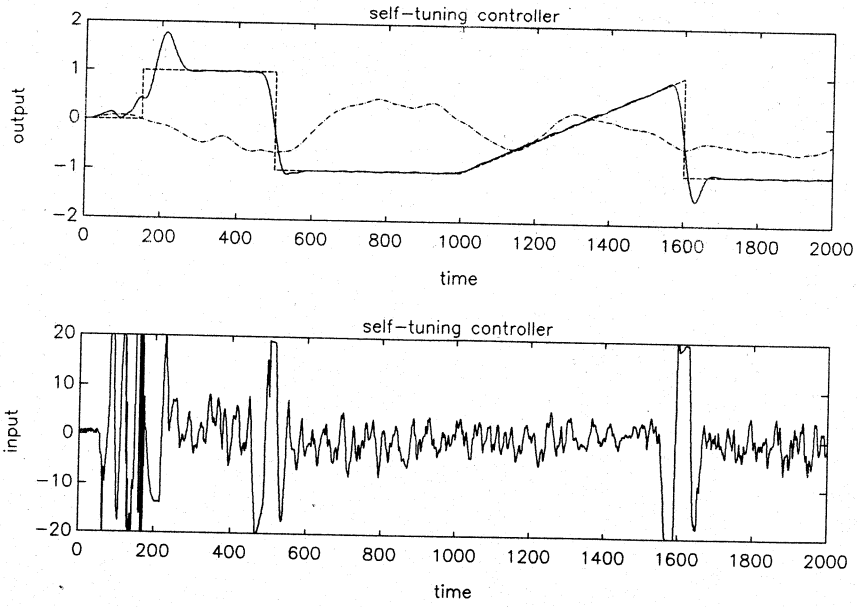

Fig. 2. The behaviour of the self-tuning controller

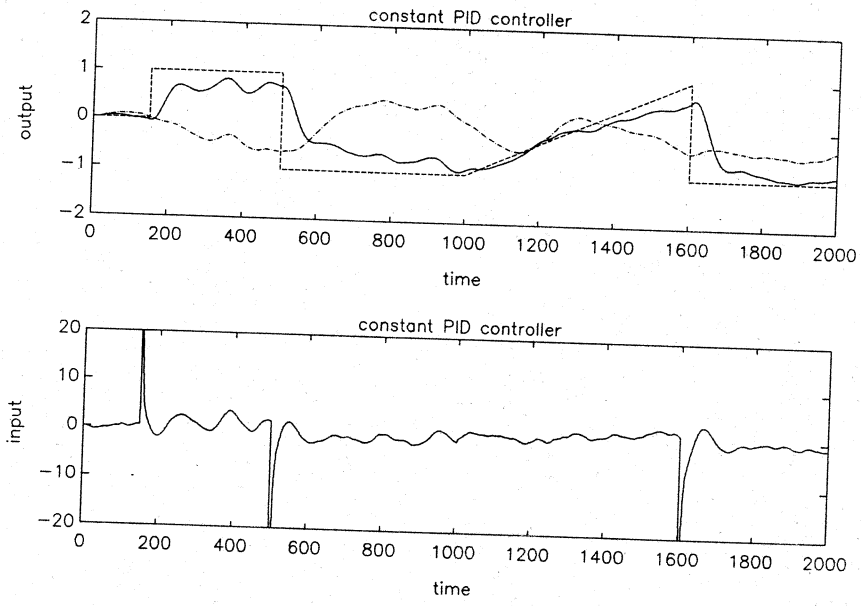

Fig. 3. The behaviour of the PID controller