

# On Steady State Minimum Variance Control Strategy

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The paper deals with the optimal digital regulation of stationary stochastic plant with single input and single output. It is assumed that the input-output relation is described by a linear difference equation and that the spectral density of additive stochastic disturbances is rational. The problem is to determine the control law (the algorithm of the digital controller in feedback) so that the variance of the plant output be minimum in steady state.

## 1. INTRODUCTION

The problem we shall deal with became to be almost classical and has been considered by many authors who applied different approaches. A comprehensive discussion of the given problem including a practical application can be found in the book by Åström [1, chap. 6]. Another solution was presented by Strejc [2] who applied the Wiener technique in complex domain. Related problems have been considered by Tsytkin [3], Chang [4], Volgin [5] and others. Many remarkable and highly important results in the control theory of stochastic discrete systems have been achieved. Nevertheless some confusing statements can be repeatedly found concerning the sensitivity and stability of optimal closed control loops in the case when the process to be controlled is of nonminimum phase type and/or unstable itself. Also most of the results do not apply for some special cases such as finite memory systems etc.

After the transformation into the state space representation the given regulator problem can be solved as a special case by the more general modern stochastic control theory [1, chap. 8], [6, chap. 9]. When this theory is applied the steady state solution of the discrete version of the matrix Riccati equation has to be found. It is commonly believed that this approach is more suitable for numerical calculation on a digital computer. But the solution of the stationary Riccati equation is not unique and it turned out that to find the correct solution is sometimes not a simple task, especially when the system is of nonminimum-phase type. It should be emphasised here that the

nonminimum-phase discrete systems are encountered rather frequently and that often the continuous minimum-phase systems get the nonminimum-phase property after the discretisation.

The reasons mentioned above initiated the idea to brush up the complex domain approach and to find a solution more general than the results known up to now and bring it into a form which could be well algorithmized uniformly for all cases.

## 2. PROBLEM STATEMENT

A linear discrete stationary stochastic system with single input  $\{u(t); t \in T\}$  and single output  $\{y(t); t \in T\}$  is considered.  $T$  is the set of integers. It is assumed that the input-output relation for any  $t \in T$  is described by the following equations

$$(1) \quad y(t) = x(t) + \eta(t),$$

$$(2) \quad x(t) + \sum_{i=1}^n a_i x(t-i) = \sum_{i=0}^m b_i u(t-k-i),$$

where  $\{\eta(t); t \in T\}$  is a weakly stationary sequence of random disturbances with zero mean and the autocovariance function

$$(3) \quad \varphi_{\eta\eta}(j) = E \eta(t) \eta(t+j).$$

It is assumed that the corresponding spectral density is a rational function

$$(4) \quad \Phi_{\eta\eta}(\zeta) = \sum_{j=-\infty}^{+\infty} \varphi_{\eta\eta}(j) \zeta^j = \sigma_e^2 \frac{\beta(\zeta) \beta(\zeta^{-1})}{\alpha(\zeta) \alpha(\zeta^{-1})}$$

where  $\beta(\zeta)$  and  $\alpha(\zeta)$  are polynomials with real coefficients

$$(5) \quad \beta(\zeta) = 1 + \beta_1 \zeta + \beta_2 \zeta^2 + \dots + \beta_\mu \zeta^\mu,$$

$$(6) \quad \alpha(\zeta) = 1 + \alpha_1 \zeta + \alpha_2 \zeta^2 + \dots + \alpha_\nu \zeta^\nu$$

and  $\sigma_e^2$  is a constant not necessarily known. The polynomials  $\beta(\zeta)$  and  $\alpha(\zeta)$  in (4) can be always chosen to have no roots inside the unit circle. As the stochastic process is assumed to be stationary,  $\alpha(\zeta)$  in addition cannot have any root on the unit circle [7].

The degrees  $\mu$  and  $\nu$  of the polynomials (5) and (6) as well as  $n$  and  $m$  in (2) may be any finite nonnegative integers.

Due to the physical realizability the discrete closed loop must contain at least one step delay. Let us include this necessary one step delay into the system to be controlled and thus in (2) always  $k \geq 1$ . Then the physically realizable linear control law does not need to contain any time delay and can be written in the following form

$$(7) \quad u(t) = - \sum_{i=1}^N r_i u(t-i) - \sum_{i=0}^M s_i y(t-i).$$

The problem is to find the lowest possible integers  $N, M$  and all coefficients in the control law (7) that 221

- a) the output variance  $E y^2(t)$  be minimum in steady state and,
- b) the closed control loop be stable.

### 3. NOTATION

To simplify the writing the following notation will be used for polynomials

$$d = d(\zeta) = d_0 + d_1 \zeta + \dots + d_\lambda \zeta^\lambda,$$

$$\|d\| \quad \text{degree of the polynomial } d; \|d\| = \lambda,$$

$$\bar{d} = d(\zeta^{-1}),$$

$$\bar{d} \quad \text{polynomial reciprocal to } d \text{ and normalized to } \bar{d}(0) = 1$$

$$\bar{d}(\zeta) = \frac{1}{d_\lambda} \zeta^\lambda d(\zeta^{-1}) = 1 + \frac{d_{\lambda-1}}{d_\lambda} \zeta + \frac{d_{\lambda-2}}{d_\lambda} \zeta^2 + \dots + \frac{d_0}{d_\lambda} \zeta^\lambda,$$

$$d^+ \quad \text{factor containing all zeros of } d \text{ lying outside and on the unit circle in the } \zeta\text{-plane normalized so that } d^+(0) = 1,$$

$$d^- \quad \text{factor of } d \text{ containing all zeros of } d \text{ lying inside the unit circle and fulfilling the relation}$$

$$d^- d^+ = d,$$

$$d^* = d^+ \bar{d}^- \text{ where } \bar{d}^- \text{ is the polynomial reciprocal to } d^- \text{ and normalized to } \bar{d}^-(0) = 1,$$

$$\zeta = z^{-1} \quad \text{for convenience the complex parameter } \zeta \text{ is used instead of customary } z \text{ in } z\text{-transform.}$$

Notice that  $d^-(\zeta) = d_0$  and  $d^*(\zeta) = d^+(\zeta)$  if the polynomial  $d(\zeta)$  has no zeros inside the unit circle.

Notice also that  $d^*(\zeta)$  has no zeros inside the unit circle and always  $d^*(0) = d_0^* = 1$ . Also  $\|d^*\| = \|d\|$  if  $d_0 \neq 0$ .

### 4. MAIN RESULT

It is well known that the steady state output variance  $E y^2(t) = \sigma_y^2$  of the stable control loop described by (1), (2), (4) and (7) can be expressed as the complex integral

$$(8) \quad \sigma_y^2 = \sigma_c^2 \frac{1}{2\pi j} \oint W(\zeta) W(\zeta^{-1}) \frac{d\zeta}{\zeta}$$

222 where the path of integration is the unit circle and

$$(9) \quad W(\zeta) = \frac{\beta}{\alpha} \frac{ar}{ar + \zeta^k bs}$$

$\alpha$  and  $\beta$  being the polynomials (5), (6) and

$$(10) \quad a = a(\zeta) = 1 + a_1\zeta + \dots + a_n\zeta^n,$$

$$(11) \quad b = b(\zeta) = b_0 + b_1\zeta + \dots + b_m\zeta^m,$$

$$(12) \quad r = r(\zeta) = 1 + r_1\zeta + \dots + r_N\zeta^N,$$

$$(13) \quad s = s(\zeta) = s_0 + s_1\zeta + \dots + s_M\zeta^M.$$

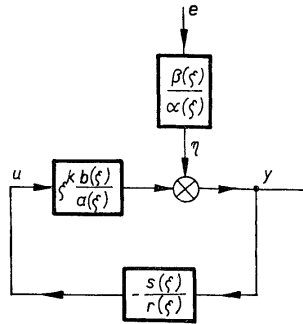


Fig. 1.

The function  $W(\zeta)$  can be interpreted as the pulse-transfer-function between the signal  $e$  and  $y$  in the block diagram shown in Fig. 1. Then the complex integral in (8) is the sum of squares of the corresponding pulse response.

To solve the problem formulated in section 2 the polynomials  $r(\zeta)$  and  $s(\zeta)$  are to be found, which minimize the integral (8) under the restriction that the closed control loop must be stable. The minimization procedure will be performed by decomposition of the integral in (8) into two terms

$$(14) \quad I = \frac{1}{2\pi j} \oint W(\zeta) W(\zeta^{-1}) \frac{d\zeta}{\zeta} = I_1 + I_2.$$

The first term  $I_1$  will be chosen so that it cannot be influenced by any stable regulation and thus represents the absolute minimum of  $I$ , while  $I_2$  is the term which can be fully compensated. To reach this situation the following rearrangement of the function  $W(\zeta)$  is suitable

$$(15) \quad W(\zeta) = \frac{\beta}{\alpha} \frac{ar + \zeta^k bs - \zeta^k bs}{ar + \zeta^k bs} = \frac{\beta}{\alpha} - \frac{\beta}{\alpha} \frac{\zeta^k bs}{ar + \zeta^k bs} =$$

$$= \zeta^k b^- \left[ \frac{\beta}{\zeta^k b^- \alpha} - \frac{\beta b^+ s}{\alpha(ar + \zeta^k bs)} \right] = \zeta^k \frac{a^- b^-}{\tilde{a}^- \tilde{b}^-} \left[ \frac{\tilde{a}^- \tilde{b}^- \beta}{a^- \zeta^k b^- \alpha} - \frac{\tilde{a}^- \beta b^+ s}{a^- \alpha(ar + \zeta^k bs)} \right].$$

The last operation in (15) is a trick, the sense of which will be apparent later on. The first term in the brackets in (15) can be further decomposed into two terms

$$(16) \quad \frac{\tilde{a}^- \tilde{b}^- \beta}{\zeta^k a^- b^- \alpha} = \frac{p}{\zeta^k b^-} + \frac{q}{a^- \alpha}.$$

This partial fractioning leads to the polynomial equation

$$(17) \quad \tilde{a}^- \tilde{b}^- \beta = a^- \alpha p + \zeta^k b^- q.$$

As no restriction concerning the degrees of the polynomials  $\beta$  and  $\alpha$  were accepted, the decomposition (16) may be not unique if  $\mu > \nu$ . Therefore the degree at least of one of the unknown polynomials must be specified. It will be seen that the minimal possible degree of  $p$

$$(18) \quad \|p\| = \kappa + k - 1,$$

$$(19) \quad \kappa = \|b^-\|$$

is the only correct choice. Notice that always  $p(0) = p_0 = 1$ .

After the decomposition (16) the function  $W(\zeta)$  (15) can be rewritten into the final form

$$(20) \quad W(\zeta) = \zeta^k \frac{a^- b^-}{\tilde{a}^- \tilde{b}^-} \left[ \frac{p}{\zeta^k b^-} + \Psi \right]$$

where

$$\Psi = \frac{q}{a^- \alpha} - \frac{\tilde{a}^- \beta b^+ s}{a^- \alpha(ar + \zeta^k bs)} = \frac{qar + (\zeta^k bq - \tilde{a}^- \beta \tilde{b}^- b^+)s}{a^- \alpha(ar + \zeta^k bs)}$$

and using (17)

$$(21) \quad \Psi = \frac{a^+ qr - \alpha p b^+ s}{\alpha(ar + \zeta^k bs)}.$$

The expression (20) has to be substituted into (8). Before doing it notice that

$$(22) \quad \frac{a^- b^-}{\tilde{a}^- \tilde{b}^-} \frac{\tilde{a}^- \tilde{b}^-}{a^- b^-} = K$$

which is a constant. This is the sense of the trick applied in the last expression of (15).

Now the integral (14) can be rewritten into the following form

$$(23) \quad I = K \frac{1}{2\pi j} \oint \frac{p\bar{p}}{b^-\bar{b}^-} \frac{d\zeta}{\zeta} + K \frac{1}{2\pi j} \oint \Psi \bar{\Psi} \frac{d\zeta}{\zeta} + 2K \frac{1}{2\pi j} \oint \frac{\bar{p}}{\zeta^{-k}\bar{b}^-} \Psi \frac{d\zeta}{\zeta}.$$

The function  $\Psi(\zeta)$  must be holomorphic inside and on the unit circle for any stable regulation. Therefore the third integral in (23) vanishes if the degree of the polynomial  $p$  is chosen according to (18)

$$K \frac{1}{2\pi j} \oint \frac{\bar{p}}{\zeta^{-k}\bar{b}^-} \Psi \frac{d\zeta}{\zeta} = b_x^- \frac{1}{2\pi j} \oint \frac{\bar{p}}{\bar{b}^-} \Psi \frac{d\zeta}{\zeta} = 0.$$

Thus the desired decomposition (14) is achieved with

$$(24) \quad I_1 = K \frac{1}{2\pi j} \oint \frac{p\bar{p}}{b^-\bar{b}^-} \frac{d\zeta}{\zeta}$$

and

$$(25) \quad I_2 = K \frac{1}{2\pi j} \oint \Psi \bar{\Psi} \frac{d\zeta}{\zeta}.$$

The term  $\sigma_e^2 I_1$  is the unreducible part of the output variance (8) while  $\sigma_e^2 I_2$  can be fully compensated by the choice

$$(26) \quad \frac{s}{r} = \frac{a^+ q}{\alpha p b^+}$$

as it follows from (21).

To determine the optimal regulator (26) it is not necessary to know the polynomials  $p$  and  $b^+$  separately. Only their product

$$(27) \quad v = p b^+$$

is required. After multiplication by  $b^+ a^+$  the polynomial equation (17) gets the form

$$(28) \quad a^* b^* \beta = \alpha x v + \zeta^k b s,$$

$$(29) \quad \|v\| = m + k - 1$$

which makes it possible to calculate the polynomial  $v$  directly. Then

$$(30) \quad r = \alpha v.$$

To check the stability of the optimal control loop let us calculate the characteristic polynomial for the optimal control law (26):

$$(31) \quad \alpha r + \zeta^k b s = a^* b^* \beta^*.$$

From this polynomial it can be seen that the optimum lies at the stability boundary if any of the polynomials  $a$ ,  $b$  or  $\beta$  has some of its zeros on the unit circle. Otherwise the optimum lies always inside the region of stability.

The result can be summarized in the following theorem.

**Theorem 1.** Consider the weakly stationary stochastic system described by the equations (1), (2), (4). The linear control law (7) is postulated.

The coefficients of the control law minimizing the output variance in steady state are given by the transfer function

$$(32) \quad \frac{s}{r} = \frac{s}{\alpha v}$$

where the polynomials  $v$  and  $s$  are determined by the polynomial equation

$$(28) \quad a^* b^* \beta = \alpha v + \zeta^k b s$$

where

$$(29) \quad \|v\| = m + k - 1.$$

When the polynomials  $a$ ,  $b$  or  $\beta$  have some zeros on the unit circle the optimum lies at the stability boundary, otherwise the optimum lies always inside the region of stability.

*Remark 1.* Notice that the polynomial equation (28) does not contain  $b^+$  and  $\bar{b}^-$  as well as  $a^+$  and  $\bar{a}^-$  separately but only their product  $b^* = b^+ \bar{b}^-$  and  $a^* = a^+ \bar{a}^-$  which can be calculated from given  $b$  and  $a$  without the knowledge of the roots by a simple iterative procedure described in [8] and [9]. The polynomial equation (28) or (17) can be most efficiently solved by algorithms for contraction of rational function developed by Nekolný [10]. The algorithms mentioned give the possibility to write a relatively simple computer program which solves numerically the given problem uniformly for all possible cases [11].

For by hand calculation the polynomial equation (17) and the relation (26) may be more suitable. However, then the full factorization of the polynomials is required.

*Remark 2.* Notice that the order of the optimal controller can be reduced if the polynomials  $a$  and  $\alpha$  have some common factor, which can be cancelled in (32) and on both sides of the polynomial equation (28).

The same situation occurs if the polynomials  $a$  and  $b$  have some common factor with zeros lying outside the unit circle.

Similarly, if  $\alpha$  and  $\beta$  have some common factor it appears also in  $s$  as it can be seen from (28) and can be cancelled in (32), too.

Therefore it is advisable to cheque the rational function (32) for common factors if they are not apriori known.

*Remark 3.* If the optimum lies near to or at the stability boundary even very small changes of parameters in unsuitable direction may cause the unstability of control loop. Therefore it is advisable to check the stability and the output-variance in the neighbourhood of the optimum and to change the adjustment to a suboptimal but less sensitive point if desired. The next simple example is given for illustration.

**Example 1.** Consider the process described by the stochastic difference equation

$$y(t) - 0.7y(t - 1) = u(t - 1) + u(t - 2) + e(t) + 0.95e(t - 1)$$

where  $\{e(t), t \in T\}$  is a stationary sequence of uncorrelated random variables with zero mean and  $\sigma_e^2 = 1$ . The corresponding polynomials are

$$\begin{aligned} a(\zeta) = a^+(\zeta) = \alpha(\zeta) &= 1 - 0.7\zeta, \\ b(\zeta) = b^+(\zeta) = b^*(\zeta) &= 1 + \zeta, \quad \bar{b}^-(\zeta) = 1, \\ \beta(\zeta) &= 1 + 0.95\zeta, \\ k &= 1. \end{aligned}$$

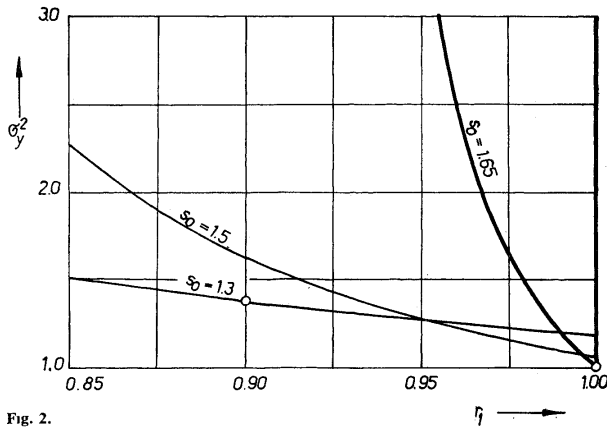


Fig. 2.

According to (18)  $\|p\| = 0$  and thus  $p(\zeta) = 1$ . The polynomial equation (17)

$$1 + 0.95\zeta = 1 - 0.7\zeta + \zeta q_0$$

gives  $q_0 = 1.65$  and the pulse transfer function (26) of the optimal controller is

$$\frac{s(\zeta)}{r(\zeta)} = \frac{s_0}{1 + r_1\zeta} \quad \text{with } s_0 = 1.65, \quad r_1 = 1.$$



As the polynomial  $b(\zeta)$  has a zero on the unit circle the optimum lies at the stability boundary. It can be seen in Fig. 2, where the output variance of the regulated process is plotted for different  $r_1$  and  $s_0 = 1.65$ . The control loop is unstable even for the smallest deviations of  $s_0$  from the value  $s_0 = 1.65$  if  $r_1 = 1$ . To show the situation more clearly also curves for other fixed  $s_0$  (Fig. 2) and  $r_1$  (Fig. 3) are plotted. The

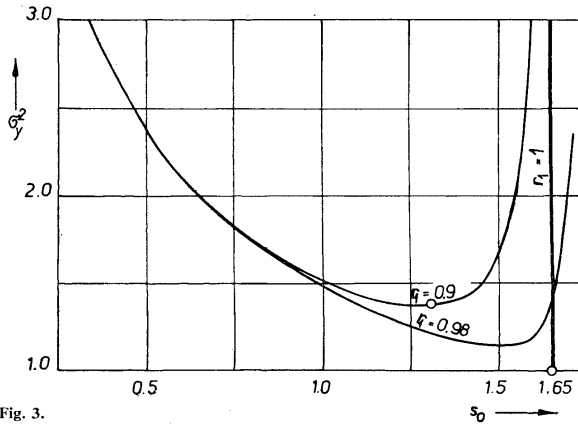


Fig. 3.

diagrams show that a suboptimal adjustment of the controller can be found, which is not very close to the stability boundary and the corresponding increase of the output variance is well acceptable. For instance  $r_1 = 0.9$  and  $s_0 = 1.3$  give the output variance  $\sigma_y^2 = 1.38$  while the absolute minimum is 1 and the output variance without control was 6.34.

### 5. IMPORTANT SPECIAL CASES

Various models can be used for description of stochastic controlled processes [12]. In this section the Theorem 1 will be applied for the most important of them.

#### Impulse response model

Consider the stochastic process described by the equation of the following form

$$(33) \quad y(t) = \sum_{i=1}^L g_i u(t-i) + \sum_{i=0}^{\lambda} h_i e(t-i).$$

The variance of the discrete white noise  $e(t)$  can be chosen so that  $h_0 = 1$ .

The coefficients  $g_i, i = 1, 2, \dots, L$  can be interpreted as the ordinates of the pulse response of a finite memory deterministic system (Fig. 4) with the pulse transfer function

$$(34) \quad g(\zeta) = g_1\zeta + g_2\zeta^2 + \dots + g_L\zeta^L.$$

Similarly the coefficients  $h_i$  are the ordinates of the pulse response of the finite memory filter with the pulse transfer function

$$(35) \quad h(\zeta) = 1 + h_1\zeta + \dots + h_k\zeta^k.$$

The filter represents the second order statistical properties of the stationary additive noise  $\eta$  in Fig. 4 and the polynomial  $h(\zeta)$  can be always chosen so that it has no zeros inside the unit circle.

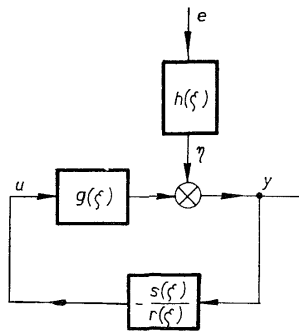


Fig. 4.

If the system has a time delay of  $k$  steps ( $k > 1$ ), then  $g_i = 0$  for  $i = 1, 2, \dots, k - 1$ . The application of the Theorem 1 to this case gives

**Corollary 1.** *The coefficients of the linear control law (7) minimizing the output variance of the stochastic system described by impulse response model (33) are determined by the polynomial equation*

$$(36) \quad g^*h = r + gs.$$

where

$$(37) \quad \|r\| = L - 1.$$

**Regression model**

Stationary stochastic processes are often described by the regression model

$$(38) \quad y(t) = \sum_{i=1}^N A_i y(t-i) + \sum_{i=1}^M B_i u(t-i) + e(t).$$

If the polynomials

$$(39) \quad A(\zeta) = 1 - A_1 \zeta - A_2 \zeta^2 - \dots - A_N \zeta^N,$$

$$(40) \quad B(\zeta) = B_1 \zeta + B_2 \zeta^2 + \dots + B_M \zeta^M$$

are introduced then the optimal control law is given by

**Corollary 2.** *If the stochastic process is described by the regression model (38) the steady state minimum variance control strategy is determined by the polynomial equation*

$$(41) \quad B^* = Ar + Bs$$

with

$$(42) \quad \|r\| = M - 1.$$

**Åström model**

The model introduced by Åström and Bohlin [13], [14] became very popular in practical application

$$(43) \quad y(t) + \sum_{i=1}^N A_i y(t-i) = \sum_{i=0}^M B_i u(t-k-i) + e(t) + \sum_{i=1}^N C_i e(t-i).$$

The coefficients  $C_i$ ,  $i = 1, 2, \dots, N$  are chosen so that the polynomial

$$(44) \quad C(\zeta) = 1 + C_1 \zeta + \dots + C_N \zeta^N,$$

has no zeros inside the unit circle.

Introduce also the polynomials

$$(45) \quad A(\zeta) = 1 + A_1 \zeta + \dots + A_N \zeta^N,$$

$$(46) \quad B(\zeta) = B_0 + B_1 \zeta + \dots + B_M \zeta^M$$

When the common factors in both equations (28) and (32) are considered ( $a = A$ ,  $\alpha = A^*$ ,  $b = B$ ,  $\beta = C$ ) the Theorem 1 gives

**Corollary 3.** *The process described by the stochastic difference equation (43) is considered. The polynomial  $C(\zeta)$  has no zeros inside the unit circle and  $B_M \neq 0$ . The linear control law (7) minimizing the output variance in steady state is determined by the polynomial equation*

$$(47) \quad B^*C = Ar + \zeta^k B_s$$

with

$$(48) \quad \|r\| = M + k - 1.$$

**Remark 4.** Notice that the regression model is a special case of Åström model (43) for  $C(\zeta) = 1$ .

Notice also the interesting fact that instability of the plant does not make any exception when the Åström model is applied. The simple equation (47) is valid generally.

The classical model considered in our problem formulation can be recalculated into the Åström form using the following relations  $A = ax$ ;  $B = bx$ ;  $C = a^*\beta$ . The calculation of  $C$  contains already the necessary factorisation of  $a$ . The characteristic polynomial of the closed control loop is  $C(\zeta) B^*(\zeta)$ . All this shows that the stability of the plant actually does not play any role but it must be considered in the classical model only for the correct calculation of the polynomial  $C$ . From this point of view the Åström model appears to be a very practical and general representation of stochastic systems.

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## VÝTAH

## K strategii regulace minimalizující rozptyl v stacionárním stavu

VÁCLAV PETERKA

Článek se zabývá číslicovou regulací stacionární stochastické soustavy s jedním vstupem a jedním výstupem. Předpokládá se, že stochastickou soustavu lze rozdělit na deterministickou část popsanou lineární diferencí rovnicí a aditivní šum s racionálním spektrem. Řeší se problém určení optimálního algoritmu číslicového regulátoru ve zpětné vazbě, který minimalizuje rozptyl regulované veličiny v ustáleném stavu. Uvádí se řešení tohoto problému, které je jednak obecnější než dosud známá řešení a jednak je snadno algoritmizovatelné. Hlavní výsledek je shrnut do následující věty.

**Věta 1.** *Uvažuje se slabě stacionární stochastická soustava popsaná rovnicemi (1), (2) a (4). Předpokládá se lineární zákon regulace (7). Koefficienty optimálního zákona regulace, který minimalizuje rozptyl výstupu v ustáleném stavu, jsou dány diskrétním přenosem číslicového regulátoru*

$$(32) \quad \frac{s}{r} = \frac{s}{\alpha v},$$

kde neznámé polynomy  $v$  a  $s$  jsou určeny polynomiální rovnicí

$$(28) \quad a^*b^*\beta = \alpha xv + \zeta^k bs,$$

232 přičemž stupeň polynomu  $v$  je

$$(29) \quad \|v\| = m + k - 1.$$

Leží-li některý kořen polynomu  $a$ ,  $b$  nebo  $\beta$  na jednotkové kružnici, leží optimum na hranici stability. Jinak leží optimum vždy uvnitř oblasti stability.

V odstavci 5 se uvádějí aplikace tohoto obecného výsledku na některé důležité speciální případy (popis soustavy impulsní charakteristikou konečné délky, regresní model, Åströmův model).

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