Multiple Model Approach to Multi-Parametric Model Predictive Control of a Nonlinear Process – simulation case study

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Abstract — The paper presents evaluation of multi-parametric multiple-model predictive control (mp-MMPC) of nonlinear process using multiple linearized models. With this approach, the nonlinear process is approximated by a set of local linear dynamics since compared to single linear model based MPC, a performance improvement is expected with the reduction of plant-to-model mismatch. Recently developed methods of multi-parametric model predictive control (mp-MPC) for hybrid systems provide an interesting opportunity for solving a class of nonlinear control problems. However the full-featured tracking hybrid mp-MPC controller results in enormous off-line computation burden. In this paper a multiple-model (MM) approach is used to reduce the optimization problem. It is evaluated in a case study, where an output feedback, offset-free tracking mp-MMPC controller was considered as a replacement for a PID controller based scheme for control of pressure in a wire annealing machine. The evaluation was carried out on a nonlinear model of the process.

I. INTRODUCTION

MODEL predictive control (MPC), also known as receding horizon control, has grown to the level of a strong player in industrial applications where linear systems subject to linear inequality constraints are involved. Nevertheless, industrial processes are in general nonlinear by their nature and operate over a broad range of operating conditions.

A common strategy in dealing with the complexity of nonlinear systems is the use of hybrid and multiple model/controllers and this way various approaches were developed. In the recent years considerable research was focused on MPC methods for hybrid systems [1, 2]. On the other hand many efforts were put in development and application of multiple model/controller solutions within the MPC field [3, 4, 5, 6, 7]. Some other methods relating to multiple-model approach to control of nonlinear systems involve gain scheduling [8, 9], multi model adaptive control [10], supervisory control [11, 12].

Following the appearance of MPC methods for hybrid systems, comprising continuous dynamic components and logical discontinuous components [1, 2], and linear model based mp-MPC methods [13, 14], multi-parametric variants of MPC methods for hybrid systems have appeared [15, 16, 17, 18, 19, 20, 21]. Also several software toolboxes have been created [22, 23, 24]. Typically, such hybrid mp-MPC methods rely on multi-parametric mixed-integer linear or quadratic programming solvers (mp-MILP/MIQP) [25, 26, 27], whereas the related on-line hybrid MPC methods employ conventional MILP or MIQP solvers.

With the explicit multi-parametric formulation of the control problem, the optimization problem may be solved in advance, off-line. This allows very simple implementation of the on-line controller in the form of a table look-up, avoiding the need for on-line optimization. Therefore it is possible to implement them in industry standard programmable logic controllers, embedded controllers, and even on FPGA chips [28].

However due to the parametric explosion of the off-line mp-MILP/MIQP computation burden with the problem dimensions, this approach is practically feasible for MPC problems of relatively small sizes. Therefore it is more interesting for rather low-level control applications, such as advanced PID replacement and small-scale multivariate processes than the conventional application niche of MPC. Essentially the use of hybrid model in the controller extends optimization problem further and consequently increases computation time to an amount that is inconvenient for practical use in industrial applications.

The presented multiple-model predictive control (MMPC) approach offers a suboptimal alternative to hybrid MPC methods that significantly reduces computation burden. Thus it provides a practically usable approach for solving nonlinear control problems by approximating the nonlinear system with a piecewise-affine (PWA) hybrid model [39]. In [40], such an approach was studied recently, with focus on stability of the hybrid control system.

The theoretical papers on hybrid mp-MPC mostly focus on the state feedback problem and related stability, feasibility and computational efficiency issues. However, in most practical applications output feedback tracking controllers with steady-state offset elimination are required [29, 30, 31, 32, 33]. A hybrid state estimator is required for output feedback [34, 35, 36, 37]. The controller-estimator interplay must not be underestimated, as it is well known that despite favourable properties of both, the joint control system may exhibit arbitrarily poor stability margins and robustness [38].

This paper presents a simulation case study. An output feedback 2-norm finite horizon mp-MMPC controller is evaluated for offset-free tracking control of pressure in the
vacuum chamber of a wire annealing machine, a high-order nonlinear system approximated with a 2nd order nonlinear model. The plant is non-square with 2 inputs 1 output. For output feedback, a switching Kalman filter (KF) is used; the active model is determined directly from the control signals and is used also to select the active controller. Additional attention is paid to implementation issues that are important in low-level control: efficient disturbance rejection, robustness to modelling error [41, 42]. Particularly to hybrid models, predictable behaviour at switching among the dynamics is required, as discontinuities in the model may lead to formally correct results that are not useful in practice.

The following sections contain: a brief overview of mp-MMPC control scheme; tracking and steady-state offset removal issues; plant description; the simulation model of the plant, the controller PWA model; controller and KF tuning; simulation results; discussion and conclusions.

II. THE MP-MMPC CONTROL SCHEME

For control the process is approximated with s linear affine models that built a hybrid PWA state space model

\[
\begin{align*}
    x(k+1) &= A_i x(k) + B_i u(k) + f_i, \quad x(k) \in P^i, \quad i = [1,\ldots,s] \\
    y(k) &= C_i x(k) + D_i u(k) + g_i 
\end{align*}
\]

where \( k \) is the discrete time index, \( A_i, B_i, C_i, D_i \) state space matrices, \( f_i, g_i \) the affine vectors, \( u \in \mathbb{R}^m \) input, \( x \in \mathbb{R}^n \) state, and \( P^i \) valid region of the state+input space in \( \mathbb{R}^{n+m} \). The system is subject to input and state constraints. For each region \( P^i \) a model exists and for it the corresponding mp-MMPC controller is designed. The currently active model is determined by Kalman filter (KF) from estimated state values. Each time step the active controller computes the control signal. The control scheme is presented in Fig. 1.

III. TRACKING IMPLEMENTATION AND OFFSET REMOVAL

The first step towards a tracking controller is offset-free output reference \( y_{ref} \) tracking. Extensions to the output cost formulation and output reference tracking are described in [14, 20, 33]. Reference tracking may be implemented by tracking velocity form model augmentation [14, 23, 33] or by direct extension of the cost function [22]. Velocity form model augmentation is also useful for specifying rate constraints on process inputs. However, the basic reference tracking controller removes steady-state offset only with non-zero reference signals.

Integral action is required for removal of steady-state offset with asymptotically non-zero disturbances [29, 30, 31, 32]. It may be achieved either by disturbance integration [24, 43, 31] or by disturbance estimation [42, 31, 32]. The estimation approach is preferred, as the integration approach is prone to integrator wind-up in case of unreachable targets, and affects the nominal performance in the absence of disturbances. Further, there is a choice of using a scheme with a target calculator (TC) or a "unified" scheme without one; the latter was selected as in [42]. An integrating disturbance estimation state \( d \) was added at the output of affine dynamics using output disturbance augmentation

\[
\begin{align*}
    x(k+1) &= \begin{bmatrix} A' & 0 \\ 0 & I \end{bmatrix} x(k) + \begin{bmatrix} B' \\ 0 \end{bmatrix} u(k) + \begin{bmatrix} f' \\ 0 \end{bmatrix} + \begin{bmatrix} G & 0 \\ 0 & I \end{bmatrix} w_r(k) \\
    y(k) &= \begin{bmatrix} C' & 1 \end{bmatrix} \begin{bmatrix} x(k) \\ d(k) \end{bmatrix} + \begin{bmatrix} I \\ D' \end{bmatrix} u(k) + g' + v(k) 
\end{align*}
\]

where \( w_r(k) = [w_{r1}(k) \ w_{r2}(k)]^T \) and \( v(k) \) are noise signals to the state and the output, respectively, and \( G \) may be used to specify the access of the noise to the state (by default, \( G = I \)).

Kalman filter was used for state estimation of the linear component of the dynamics, by assuming that a stochastic variable \( w(k) \) with covariance \( Q_{KF} \) acts the augmented state \( \begin{bmatrix} x(k) \\ d(k) \end{bmatrix} \), and that measurement noise \( v(k) \) with covariance \( R_{KF} \) at the output. Switching is performed by changing the gain \( C' \) and offset \( g' \) in the output equation, regarding to the currently active dynamic \( i \), determined from the estimated system states as described in Section VI. The corresponding (active) controller computes the next control signal vector based on its previous value.

The control problem is ill-conditioned for a plant with more inputs than outputs. With the unified scheme, this issue may be solved by using a fixed input reference value \( u_i \) with an appropriate (small) penalty \( R_u \) in the cost function for the surplus control input(s). In mp-MPC the use of fixed reference values \( u_{ref} \) via a coordinate transformation \( u = u - u_{ref} \) is beneficial in order not to increase computational demand. If the implementation does not allow \( u \) signal references [23], this may be achieved by using a state reference and penalty for the past control signal state \( u(k-1) \) within the tracking augmented state vector.

IV. PRESSURE CONTROL IN WIRE ANNEALER

The case study control problem is related to the vacuum subsystem of a wire annealing machine of Plasmat Italia GmbH. In the annealer, the processed metal wire is heated using magneto-focused plasma in an adequate inert gas atmosphere. The controller maintains the specified pressure \( p \) (process output \( y \)) in the vacuum chamber of the annealer that may vary depending on the type of wire and gas. The construction of the vacuum subsystem ensures that a certain desired pressure profile along the vacuum chamber is maintained to prevent undesired leakage. Several vacuum
pumps are connected to different chambers that are separated by sealings. The pressure $p$ is controlled roughly by adjusting the frequency converters of the pumps $u_1$ connected to chambers at wire exit (right hand side of Fig. 2). Additionally, a valve $u_2$ bypassing the sealing before the main chamber is used for fast regulation, with approximately five times faster response but a limited action range. The controller must be able to rapidly suppress fast-acting disturbances that appear during plant operation, such as momentary sealing changes, ignition of plasma, etc. It must be able to operate over a large range of operating points, affected by the pressure set-point, wire diameter, machine temperature during start-up, etc. Also, it must suppress measurement noise efficiently.

From the experimental results with linear model based mp-MPC [42] it was evident that the control performance varies with the operating point considerably; the experiments indicated that the dependence on $u_1$ was the most evident, whereas would $u_2$ mostly remain within the linear region with proper controller tuning. In addition, the static characteristic of $p$ as a function of $u_2$ is dependent on $u_1$. Within the limited time for experimentation, three operating points (OP) were examined, as displayed in Table I. There are considerable changes in local gains; changes in dynamics are also present but less expressed.

V. SIMULATION MODEL

A simple nonlinear simulation model was built for the purpose of controller evaluation in simulation. The model comprises invariant linear dynamics and static nonlinear functions at its inputs $u_{in}$. The static nonlinearity at $u_{lin}$ is a polynomial approximation of the static characteristic $y = f(u_1)$, including an appropriate offset. The static nonlinearity at $u_{lin2}$ is a two-dimensional PWA look-up table, defining $u_2$ with respect to $u_1$ according to Table I, and including offset. The static characteristic $y = f(u_2)$ is disregarded, except for additional limits set to include only the linear region: 15 < $u_2$ < 45 [%]; not much effective range can be gained outside this region in practice. Measurement noise is also included.

VI. CONTROLLER MODELS

The aim of hybrid modelling was to extend the model of the original linear mp-MPC controller with the known information regarding operating point dependent changes of gains and offset in Table I. All dynamics are based on a unity-gain linear discrete-time state-space model with two inputs and one output, with second order dynamics for each input, with sampling time $T_s = 0.2$ s.

\[
A = \begin{bmatrix} 0.7762 & -0.0600 & 0 & 0 \\ 0.1769 & 0.9937 & 0 & 0 \\ 0 & 0 & 0.4209 & -0.4622 \\ 0 & 0 & 0.1362 & 0.9473 \end{bmatrix}, \quad B = \begin{bmatrix} 0.0600 & 0 \\ 0.0063 & 0 \\ -0.4622 & 0 \\ 0.0527 & 0 \end{bmatrix}, \quad C = [0, -1, 0, -1], \quad D = [0, 0] \quad (3)
\]

For each PWA dynamic, affine gains and offsets for $u_1$ and $u_2$ model branches are assigned in the output equation

\[
y = C^T x + Du + g^T \quad (4)
\]

where $C$ and $g^T$ coefficients for particular region are given in Table II.
Initially the steady-state characteristic \( p = f(u_1, u_2) \) of the PWA model with switching with regard to \( u_1 \) were considered; the \( u_1 \) boundaries are at 11.25 Hz and 13.75 Hz. However, undesired control performance may appear around the discontinuous boundaries. For example, the controller may refuse to follow the reference signal across the region boundary if the cost of performance at the boundary (with tracking offset) is lower. Therefore, continuous boundaries are desired. Different solutions to this issue may be approached. In the upper chart of Fig. 4, the space is divided in 6 triangular planes by splitting the initial three rectangular regions. The lower chart of Fig. 4 shows the solution with the switching lines moved to static model plane intersections, while the number and the parameters of the dynamics are unaffected. The latter solution was adopted in this work in order to reduce the complexity. The plane boundaries of plane pairs (1, 2) and (2, 3) are

\[
\begin{align*}
\nu^{2(1,2)} &= \frac{(C_2(2) - C_1(2))u_1 + (g_2 - g_1)}{(C_1(4) - C_2(4))} \\
\nu^{2(2,3)} &= \frac{(C_1(2) - C_2(2))u_1 + (g_1 - g_2)}{(C_2(4) - C_1(4))}
\end{align*}
\]

Notice that continuous transition of the model states is also required; otherwise bumps may be noticed at boundaries.

### VII. CONTROLLER TUNING

For practical purposes, computation times no more than a few minutes were considered useful, therefore only short finite horizon lengths could be used. Tuning via simulation requires long computations and because the effects of the tuning parameters are not always obvious in time domain. Therefore Local linear analysis (LLA), described in [42] was used. It is based on a closed-loop system equation that describes the relations from the inputs \( w_i(k), \nu(k) \) and \( y_i(k) \) to the noise-free output \( y_d(k) \), obtained by combining the equations of the control law, the estimator and the process. Root locus diagrams of controller poles are most valuable. Tuning is first made for the unconstrained region with the nominal model from the intermediate OP and verified with models of other OPs. After the mp-MPC controller is calculated, constrained regions and sequences with varying dynamics may be analysed.

The following set of parameters was selected:
- **Linear controller and MM controllers I and II:**
  
  \( N = 6, N_u = 2, R_u = \text{diag}(0.05 0.05) \), \( R_w = \text{diag}(10^{-6} 0.02) \)
  where \( N_u \) is the control horizon.
- **MM controller III (highest gain):**
  
  \( N = 6, N_u = 2, R_u = \text{diag}(0.05 0.05), R_w = \text{diag}(10^{-6} 0.025) \)

### VIII. KALMAN FILTER FINE TUNING

Extended LLA of the closed loop system was used. Primarily, local linear dynamics in the three OPs were studied for the unconstrained controller regions. In addition to nominal dynamics, the effect of plant-to-model mismatch for a selected set of models was always examined. Root locus diagrams and frequency characteristics of the sensitivity functions were observed.

Useful results are obtained with the MPC standard output step disturbance model, with KF parameters:

\( Q_{KF} = \text{diag}(10^{-6} 10^{-6} 10^{-6} 10^{-6}), R_{KF} = 10^{-3} \),

although the bandwidth and the robustness are not as good as with linear mp-MPC with longer horizons [42].

The efficiency of this disturbance model is limited, as there is always a slow estimator pole on the real axis, so the estimation error does not vanish faster than the controller dynamics. However, only a negligible improvement could be made by adjusting other diagonal elements of \( Q_{KF} \), while the robustness to plant-to-model mismatch was unacceptable with the input disturbance model. Faster estimator dynamics may also be achieved by using pole placement, however all such attempts resulted in high estimator gains and were oversensitive to plant-to-model mismatch.

### IX. SIMULATION STUDY

Fig. 5 shows tracking of a "staircase" \( y_1 \) signal across the
relevant operating range of the process, including all three OPs (PWA dynamics), with both MM and linear model based mp-MPC with the same tuning parameters. Tracking is offset-free with both controllers. Effects of plant-to-model mismatch can be observed in linear mp-MPC response (black): overshoot is present at higher $y_r$ values where the process gain is higher, while the response gets sluggish at lower $y_r$ values. These effects are reduced with the MM mp-MPC controller; however, the response with the same tuning parameters is not expected to look the same all operating points due to different model dynamics, and a (small) degree of modelling mismatch is still present. At some points, sluggish response due to transient $u_2$ saturations may be noticed with both controllers, particularly at step changes of $y_r$ when $u_2$ has not settled to its set-point 30%.

In Fig. 6 and Fig. 7 the controller performance is shown around the two extreme OPs, 2.9 mbar and 7 mbar. A sequence of step changes of $y_r$ and disturbances at $u_1$, $u_2$, and $y$ in both directions is made in 20 s intervals. The MMPC is able to achieve better performance by switching to the matching local dynamic. The difference among the controllers is relatively small; however, more difference is expected in practice due to unmodelled changes in the dynamics. Due to operation near steady state, saturations of $u_2$ do not occur.

**X. CONCLUSIONS**

In the paper a slight advantage of MM approach over linear controller due to decreased plant-to-model mismatch. The performance is similar to hybrid mp-MPC approach [44], however here significantly (100+ times) computation is required, which allows use of longer horizons, where the performance improvement over linear model mp-MPC becomes more pronounced. Besides, further improvement of MM controller in comparison to linear model based approach is expected in plant experimental test due to additional modelling error.

Another advantage of MM approach is that it allows different tuning parameters to be used for each controller, enabling us to equalize the response among local controller models with different static gain and to maximize control performance and robustness as suggested in [42].

However, splitting a hybrid mp-MPC into several linear mp-MPC controllers causes the loss of optimality, as
switching has to be based on external parameter. Thus neither the optimal cost, nor the switch is foreseen in the individual controller's prediction. Therefore current efforts and further work are focused in simplifications of the hybrid mp-MPC.

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REFERENCES