Multiple Model Approach to Multi-Parametric Model Predictive Control of a Nonlinear Process – simulation case study

Boštjan Pregelj, and Samo Gerkšič

Abstract — **The paper presents evaluation of multi-parametric multiple-model predictive control (mp-MMPC) of nonlinear process using multiple linearized models. With this approach, the nonlinear process is approximated by a set of local linear dynamics since compared to single linear model based MPC, a performance improvement is expected with the reduction of plant-to-model mismatch. Recently developed methods of multi-parametric model predictive control (mp-MPC) for hybrid systems provide an interesting opportunity for solving a class of nonlinear control problems. However the full-featured tracking hybrid mp-MPC controller results in enormous off-line computation burden. In this paper a multiple-model (MM) approach is used to reduce the optimization problem. It is evaluated in a case study, where an output feedback, offset-free tracking mp-MMPC controller was considered as a replacement for a PID controller based scheme for control of pressure in a wire annealing machine. The evaluation was carried out on a nonlinear model of the process.**

I. Introduction

MODEL predictive control (MPC), also known as receding horizon control, has grown to the level of a strong player in industrial applications where linear systems subject to linear inequality constraints are involved. Nevertheless, industrial processes are in general nonlinear by their nature and operate over a broad range of operating conditions.

A common strategy in dealing with the complexity of nonlinear systems is the use of hybrid and multiple model/controllers and this way various approaches were developed. In the recent years considerable research was focused on MPC methods for hybrid systems [1, 2]. On the other hand many efforts were put in development and application of multiple model/controller solutions within the MPC field [3, 4, 5, 6, 7]. Some other methods relating to multiple-model approach to control of nonlinear systems involve gain scheduling [8, 9], multi model adaptive control [10], supervisory control [11, 12].

Following the appearance of MPC methods for hybrid systems, comprising continuous dynamic components and logical discontinuous components [1, 2], and linear model based mp-MPC methods [13, 14], multi-parametric variants of MPC methods for hybrid systems have appeared [15, 16, 17, 18, 19, 20, 21]. Also several software toolboxes have been created [22, 23, 24]. Typically, such hybrid mp-MPC methods rely on multi-parametric mixed-integer linear or quadratic programming solvers (mp-MILP/MIQP) [25, 26, 27], whereas the related on-line hybrid MPC methods employ conventional MILP or MIQP solvers.

With the explicit multi-parametric formulation of the control problem, the optimization problem may be solved in advance, off-line. This allows very simple implementation of the on-line controller in the form of a table look-up, avoiding the need for on-line optimization. Therefore it is possible to implement them in industry standard programmable logic controllers, embedded controllers, and even on FPGA chips [28].

However due to the parametric explosion of the off-line mp-MILP/MIQP computation burden with the problem dimensions, this approach is practically feasible for MPC problems of relatively small sizes. Therefore it is more interesting for rather low-level control applications, such as advanced PID replacement and small-scale multivariate processes than the conventional application niche of MPC. Essentially the use of hybrid model in the controller extends optimization problem further and consequentially increases computation time to an amount that is inconvenient for practical for use in industrial applications.

The presented multiple-model predictive control (MMPC) approach offers a suboptimal alternative to hybrid MPC methods that significantly reduces computation burden. Thus it provides a practically usable approach for solving nonlinear control problems by approximating the nonlinear system with a piecewise-affine (PWA) hybrid model [39]. In [40], such an approach was studied recently, with focus on stability of the hybrid control system.

The theoretical papers on hybrid mp-MPC mostly focus on the state feedback problem and related stability, feasibility and computational efficiency issues. However, in most practical applications output feedback tracking controllers with steady-state offset elimination are required [29, 30; 31, 32, 33]. A hybrid state estimator is required for output feedback [34, 35, 36, 37]. The controller-estimator interplay must not be underestimated, as it is well known that despite favourable properties of both, the joint control system may exhibit arbitrarily poor stability margins and robustness [38].

This paper presents a simulation case study. An output feedback 2-norm finite horizon mp-MMPC controller is evaluated for offset-free tracking control of pressure in the

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B. Pregelj is with Jozef Stefan Institute, Ljubljana, Slovenia (phone: +386 1 477 3732; fax: +386 1 477 3994; e-mail: bostjan.pregelj@ ijs.si).

S. Gerkšič is with Jozef Stefan Institute, Ljubljana, Slovenia (e-mail: samo.gerksic@ijs.si).

vacuum chamber of a wire annealing machine, a high-order nonlinear system approximated with a 2nd order nonlinear model. The plant is non-square with 2 inputs 1 output. For output feedback, a switching Kalman filter (KF) is used; the active model is determined directly from the control signals and is used also to select the active controller. Additional attention is paid to implementation issues that are important in low-level control: efficient disturbance rejection, robustness to modelling error [41, 42]. Particularly to hybrid models, predictable behaviour at switching among the dynamics is required, as discontinuities in the model may lead to formally correct results that are not useful in practice.

The following sections contain: a brief overview of mp-MMPC control scheme; tracking and steady-state offset removal issues; plant description; the simulation model of the plant, the controller PWA model; controller and KF tuning; simulation results; discussion and conclusions.

II. THE MP-MMPC CONTROL SCHEME

For control the process is approximated with s linear affine models that built a hybrid PWA state space model

$$\begin{aligned} x(k+1) &= A^i x(k) + B^i u(k) + f^i \\ y(k) &= C^i x(k) + D^i u(k) + g^i \end{aligned} \quad \text{if} \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} \in P^i,\ i = \{1,\ldots,s\} \tag{1}$$

where k is the discrete time index, A^i, B^i C^i, D^i state space matrices, f^i, g^i the affine vectors, $u \in \Re^m$ input, $x \in \Re^n$ state, and P^i valid region of the state+input space in $\Re^{n+m}$. The system is subject to input and state constraints. For each region P^i a model exists and for it the corresponding mp-MPC controller is designed. The currently active model is determined by Kalman filter (KF) from estimated state values. Each time step the active controller computes the control signal. The control scheme is presented in Fig. 1.

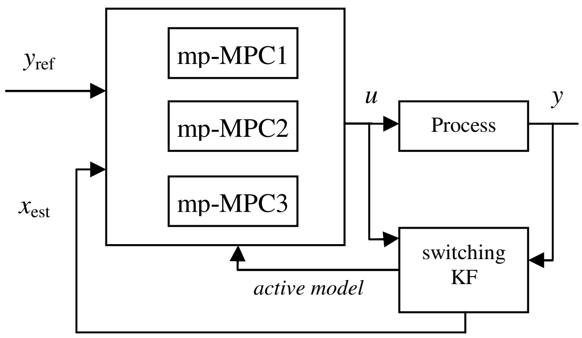

Fig. 1. Multiple-model predictive control scheme.

III. TRACKING IMPLEMENTATION AND OFFSET REMOVAL

The first step towards a tracking controller is offset-free output reference y_r tracking. Extensions to the output cost formulation and output reference tracking are described in [14, 20, 33]. Reference tracking may be implemented by tracking velocity form model augmentation [14, 23, 33] or by direct extension of the cost function [22]. Velocity form model augmentation is also useful for specifying rate constraints on process inputs. However, the basic reference tracking controller removes steady-state offset only with non-zero reference signals.

Integral action is required for removal of steady-state offset with asymptotically non-zero disturbances [29 30; 31, 32]. It may be achieved either by disturbance integration [24, 43, 31] or by disturbance estimation [42, 31, 32]. The estimation approach is preferred, as the integration approach is prone to integrator wind-up in case of unreachable targets, and affects the nominal performance in the absence of disturbances. Further, there is a choice of using a scheme with a target calculator (TC) or a "unified" scheme without one; the latter was selected as in [42]. An integrating disturbance estimation state d was added at the output of affine dynamics using output disturbance augmentation

$$\begin{bmatrix} x(k+1) \\ d(k+1) \end{bmatrix} = \begin{bmatrix} A^i & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} x(k) \\ d(k) \end{bmatrix} + \begin{bmatrix} B^i \\ 0 \end{bmatrix} u(k) + \begin{bmatrix} f^i \\ 0 \end{bmatrix} + \begin{bmatrix} G & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} w_x(k) \\ w_d(k) \end{bmatrix} \tag{2}$$

$$y(k) = \begin{bmatrix} C^i & 1 \end{bmatrix} \begin{bmatrix} x(k) \\ d(k) \end{bmatrix} + \begin{bmatrix} D^i & 0 \end{bmatrix} u(k) + g^i + v(k)$$

where $w_a(k) = [w_x^T(k)\ w_d^T(k)]^T$ and $v(k)$ are noise signals to the state and the output, respectively, and G may be used to specify the access of the noise to the state (by default, $G = I$).

Kalman filter was used for state estimation of the linear component of the dynamics, by assuming that a stochastic variable $w(k)$ with covariance Q_{KF} acts the augmented state $[x^T(k)\ \ d^T(k)]$, and that measurement noise $v(k)$ with covariance R_{KF} at the output. Switching is performed by changing the gain C^i and offset g^i in the output equation, regarding to the currently active dynamic i, determined from the estimated system states as described in Section VI. The corresponding (active) controller computes the next control signal vector based on its previous value.

The control problem is ill-conditioned for a plant with more inputs than outputs. With the unified scheme, this issue may be solved by using a fixed input reference value u_r with an appropriate (small) penalty R_u in the cost function for the surplus control input(s). In mp-MPC the use of fixed reference values u_{rf} via a coordinate transformation $u_n = u - u_{rf}$ is beneficial in order not to increase computational demand. If the implementation does not allow u signal references [23], this may be achieved by using a state reference and penalty for the past control signal state $u(k–1)$ within the tracking augmented state vector.

IV. PRESSURE CONTROL IN WIRE ANNEALER

The case study control problem is related to the vacuum subsystem of a wire annealing machine of PlasmaIt GmbH. In the annealer, the processed metal wire is heated using magneto-focused plasma in an adequate inert gas atmosphere. The controller maintains the specified pressure p (process output y) in the vacuum chamber of the annealer that may vary depending on the type of wire and gas. The construction of the vacuum subsystem ensures that a certain desired pressure profile along the vacuum chamber is maintained to prevent undesired leakage. Several vacuum pumps are connected to different chambers that are separated by sealings. The pressure p is controlled roughly by adjusting the frequency converters of the pumps u_1 connected to chambers at wire exit (right hand side of Fig. 2). Additionally, a valve u_2 bypassing the sealing before the main chamber is used for fast regulation, with approximately five times faster response but a limited action range. The controller must be able to rapidly suppress fast-acting disturbances that appear during plant operation, such as momentary sealing changes, ignition of plasma, etc. It must be able to operate over a large range of operating points, affected by the pressure set-point, wire diameter, machine temperature during start-up, etc. Also, it must suppress measurement noise efficiently.

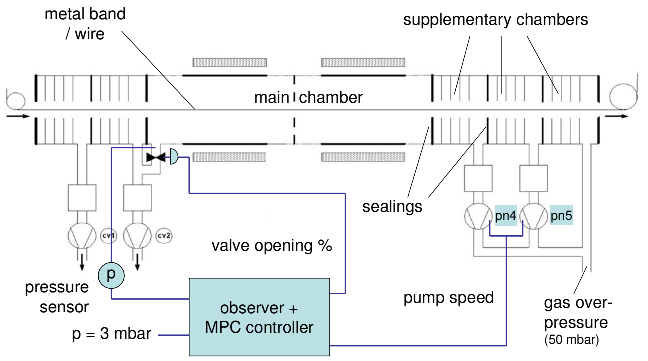

Fig. 2. Vacuum subsystem of the annealer

Due to the properties of the actuators, amplitude and rate constraints must be regarded: $0 < u_1 < 50$ [s^{-1}], $0 < u_2 < 100$ [%], $-5 < \Delta u_1 < 5$ [s^{-2}], $-50 < \Delta u_2 < 50$ [%/s]. The measurement range is $0 < p < 133$ [mbar], however the pumps at the wire input always maintain it below 22 mbar. Regarding the spare degree of freedom, which is available when the narrow u_2 constraints are inactive, it is reasonable to keep u_2 in the centre of its linear range when possible, so that the controller may efficiently react to disturbances in any direction.

Both actuators exhibit static nonlinear characteristics. Approximations of the characteristics, made from measurement data in a limited number of operating points, are shown in Fig. 3.

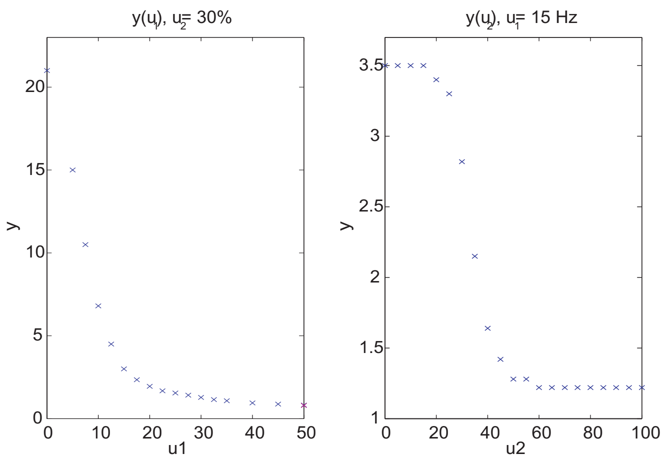

Fig. 3. Static characteristics: (left) p [mbar] as a function of u_1 when u_2 = 30 %; (right) p [mbar] as a function of u_2, when u_1 = 15 Hz

From the experimental results with linear model based mp-MPC [42] it was evident that the control performance varies with the operating point considerably; the experiments indicated that the dependence on u_1 was the most evident, whereas would u_2 mostly remain within the linear region with proper controller tuning. In addition, the static characteristic of p as a function of u_2 is dependent on u_1. Within the limited time for experimentation, three operating points (OP) were examined, as displayed in Table I. There are considerable changes in local gains; changes in dynamics are also present but less expressed.

V. Simulation Model

A simple nonlinear simulation model was built for the purpose of controller evaluation in simulation. The model comprises invariant linear dynamics and static nonlinear functions at its inputs u_{lin}. The static nonlinearity at u_{lin1} is a polynomial approximation of the static characteristic $y = f(u_1)$, including an appropriate offset. The static nonlinearity at u_{lin2} is a two-dimensional PWA look-up table, defining u_2 with respect to u_1 according to Table 1, and including offset. The static characteristic $y = f(u_2)$ is disregarded, except for additional limits set to include only the linear region: $15 < u_2 < 45$ [%]; not much effective range can be gained outside this region in practice. Measurement noise is also included.

TABLE I
Examined operation points – gains and offsets

OP	u_1 [Hz]	u_2 [%]	u_1 gain	u_2 gain
1 (low extreme)	15	30	-0.3203	-1.0057
2 (high extreme)	10	30	-1.0010	-2.4136
3 (intermediate)	12.5	30	-0.7007	-1.7096

VI. Controller Models

The aim of hybrid modelling was to extend the model of the original linear mp-MPC controller with the known information regarding operating point dependent changes of gains and offset in Table I.

All dynamics are based on a unity-gain linear discrete-time state-space model with two inputs and one output, with second order dynamics for each input, with sampling time T_s = 0.2 s

$$A = \begin{bmatrix} 0.7762 & -0.0600 & 0 & 0 \\ 0.1769 & 0.9937 & 0 & 0 \\ 0 & 0 & 0.4209 & -0.4622 \\ 0 & 0 & 0.1362 & 0.9473 \end{bmatrix}, \quad B = \begin{bmatrix} 0.0600 & 0 \\ 0.0063 & 0 \\ 0 & 0.4622 \\ 0 & 0.0527 \end{bmatrix} \tag{3}$$

$$C = [0 \quad -1 \quad 0 \quad -1], \quad D = [0 \quad 0]$$

For each PWA dynamic, affine gains and offsets for u_1 and u_2 model branches are assigned in the output equation

$$y = C^i x + Du + g^i \tag{4}$$

where C^i and g^i coefficients for particular region are given in Table II.

TABLE II

PIECEWISE AFFINE DYNAMICS GAINS AND OFFSETS

PWA dynamic (i)	output (gain) matrix C_i	offset (g_i)
1	[0 -1.0010 0 -2.4136]	4.2408
2	[0 -0.7007 0 -1.7096]	-1.2497
3	[0 -0.3203 0 -1.0057]	-8.5920

Initially the steady-state characteristic $p = f(u_1, u_2)$ of the PWA model with switching with regard to u_1 were considered; the u_1 boundaries are at 11.25 Hz and 13.75 Hz. However, undesired control performance may appear around the discontinuous boundaries. For example, the controller may refuse to follow the reference signal across the region boundary if the cost of performance at the boundary (with tracking offset) is lower. Therefore, continuous boundaries are desired. Different solutions to this issue may be approached. In the upper chart of Fig. 4, the space is divided in 6 triangular planes by splitting the initial three rectangular regions. The lower chart of Fig. 4 shows the solution with the switching lines moved to static model plane intersections, while the number and the parameters of the dynamics are unaffected. The latter solution was adopted in this work in order to reduce the complexity. The plane boundaries of plane pairs (1, 2) and (2, 3) are

$$u_{2(1,2)} = \frac{(C_2(2) - C_1(2))u_1 + (g_2 - g_1)}{(C_1(4) - C_2(4))}$$
$$u_{2(2,3)} = \frac{(C_3(2) - C_2(2))u_1 + (g_3 - g_2)}{(C_2(4) - C_3(4))} \tag{5}$$

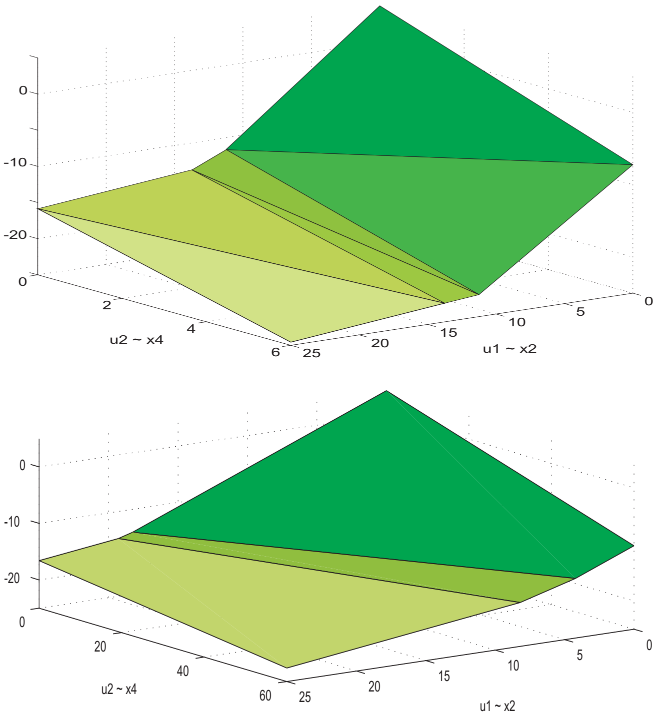

Fig. 4. Steady-state static characteristics $p = f(u_1, u_2)$ of the PWA model: (above) based on u_1 switching, further divided to avoid discontinuity; (below) more simple with switching on model plane intersection lines.

Notice that continuous transition of the model states is also required; otherwise bumps may be noticed at boundaries.

VII. CONTROLLER TUNING

For practical purposes, computation times no more than a few minutes were considered useful, therefore only short finite horizon lengths could be used. Tuning via simulation requires long computations and because the effects of the tuning parameters are not always obvious in time domain. Therefore *Local linear analysis* (LLA), described in [42] was used. It is based on a closed-loop system equation that describes the relations from the inputs $w_d(k)$, $v(k)$ and $y_r(k)$ *to* the noise-free output $y_{nf}(k)$, obtained by combining the equations of the control law, the estimator and the process. Root locus diagrams of controller poles are most valuable. Tuning is first made for the unconstrained region with the nominal model from the intermediate OP and verified with models of other OPs. After the mp-MPC controller is calculated, constrained regions and sequences with varying dynamics may be analysed.

The following set of parameters was selected:

- *Linear controller and MM controllers I and II:*

$N = 6$, $N_u = 2$, $R_{du} = \mathrm{diag}([0.1\ 0.05])$, $R_u = \mathrm{diag}([10^{-6}\ 0.02])$ where N_u is the control horizon.

- *MM controller III (highest gain):*

$N = 6$, $N_u = 2$, $R_{du} = \mathrm{diag}([0.05\ 0.05])$, $R_u = \mathrm{diag}([10^{-6}\ 0.025])$

VIII. KALMAN FILTER FINE TUNING

Extended LLA of the closed loop system was used. Primarily, local linear dynamics in the three OPs were studied for the unconstrained controller regions. In addition to nominal dynamics, the effect of plant-to-model mismatch for a selected set of models was always examined. Root locus diagrams and frequency characteristics of the sensitivity functions were observed.

Useful results are obtained with the MPC standard output step disturbance model, with KF parameters:
$Q_{KF} = \mathrm{diag}([10^{-6}\ 10^{-6}\ 10^{-6}\ 10^{-6}\ 1])$, $R_{KF} = 10^{-3}$,
although the bandwidth and the robustness are not as good as with linear mp-MPC with longer horizons [42].

The efficiency of this disturbance model is limited, as there is always a slow estimator pole on the real axis, so the estimation error does not vanish faster than the controller dynamics. However, only a negligible improvement could be made by adjusting other diagonal elements of Q_{KF}, while the robustness to plant-to-model mismatch was inacceptable with the input disturbance model. Faster estimator dynamics may also be achieved by using pole placement, however all such attempts resulted in high estimator gains and were oversensitive to plant-to-model mismatch.

IX. SIMULATION STUDY

Fig. 5 shows tracking of a "staircase" y_r signal across the relevant operating range of the process, including all three OPs (PWA dynamics), with both MM and linear model based mp-MPC with the same tuning parameters. Tracking is offset-free with both controllers. Effects of plant-to-model mismatch can be observed in linear mp-MPC response (black): overshoot is present at higher y_r values where the process gain is higher, while the response gets sluggish at lower y_r values. These effects are reduced with the MM mp-MPC controller; however, the response with the same tuning parameters is not expected to look the same all operating points due to different model dynamics, and a (small) degree of modelling mismatch is still present. At some points, sluggish response due to transient u_2 saturations may be noticed with both controllers, particularly at step changes of y_r when u_2 has not settled to its set-point 30 %.

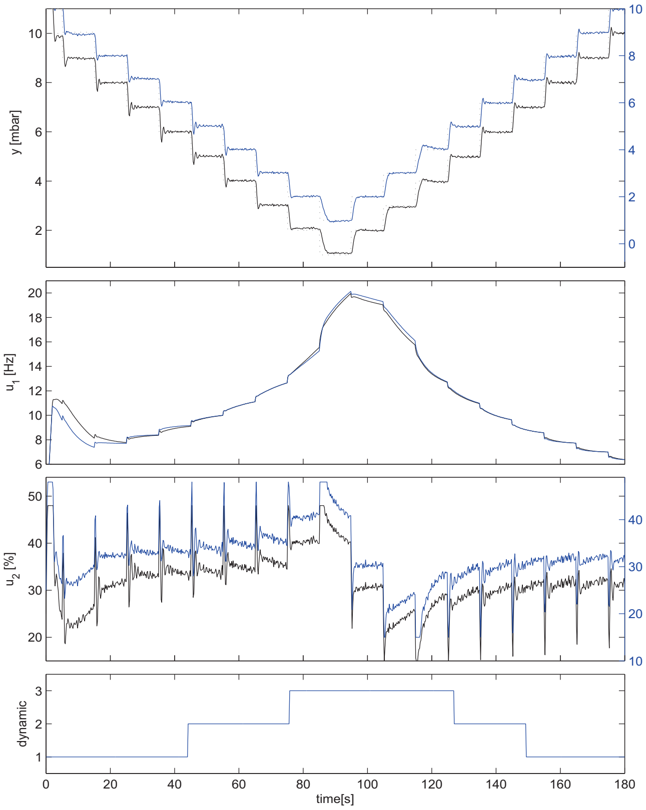

Fig. 5. Staircase y_r signal tracking across the operating range using linear (left axis, black line) and hybrid model based mp-MPC (right axis, grey). Top to bottom: y_r (dotted) and y; u_1; u_2; the active linear model i used by the switching KF.

In Fig. 6 and Fig. 7 the controller performance is shown around the two extreme OPs, 2.9 mbar and 7 mbar. A sequence of step changes of y_r and disturbances at u_1, u_2, and y in both directions is made in 20 s intervals. The MMPC is able to achieve better performance by switching to the matching local dynamic. The difference among the controllers is relatively small; however, more difference is expected in practice due to unmodelled changes in the dynamics. Due to operation near steady state, saturations of u_2 do not occur.

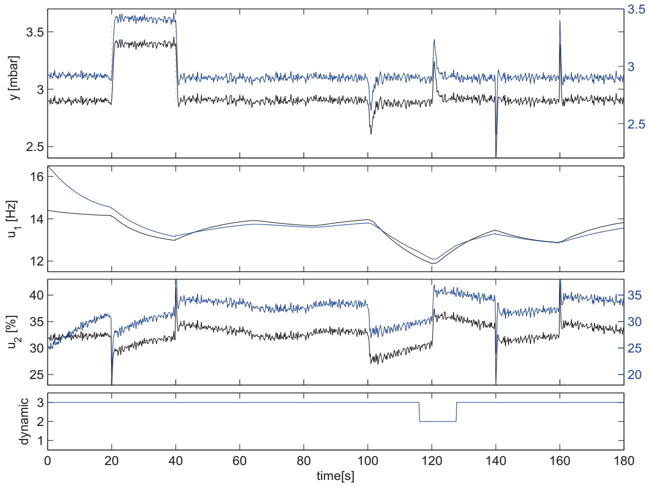

Fig. 6. Sequence of step changes of y_r and disturbances at u_1, u_2, and y using linear (left axis, black line) and hybrid model based mp-MPC (right axis, grey) at OP1, 2.9 mbar. Signals as in Fig. 5.

X. Conclusions

In the paper a slight advantage of MM approach over linear controller due to decreased plant-to-model mismatch. The performance is similar to hybrid mp-MPC approach [44], however here significantly (100+ times) computation is required, which allows use of longer horizons, where the performance improvement over linear model mp-MPC becomes more pronounced. Besides, further improvement of MM controller in comparison to linear model based approach is expected in plant experimental test due to additional modelling error.

Another advantage of MM approach is that it allows different tuning parameters to be used for each controller, enabling us to equalize the response among local controller models with different static gain and to maximize control performance and robustness as suggested in [42].

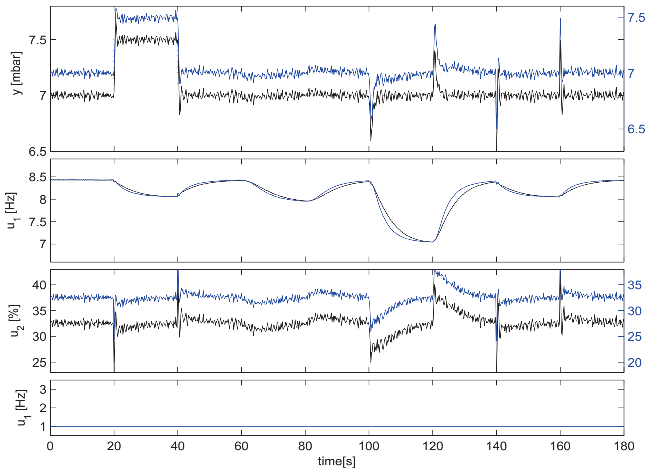

Fig. 7. Sequence of step changes of y_r and disturbances at u_1, u_2, and y using linear (left axis, black line) and hybrid model based mp-MPC (right axis, grey) at OP2, 7 mbar. Signals as in Fig. 5.

However, splitting a hybrid mp-MPC into several linear mp-MPC controllers causes the loss of optimality, as switching has to be based on external parameter. Thus neither the optimal cost, nor the switch is foreseen in the individual controller's prediction. Therefore current efforts and further work are focused in simplifications of the hybrid mp-MPC.

XI. ACKNOWLEDGEMENT

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REFERENCES

[1] A. Bemporad, and M. Morari, "Control of systems integrating logic, dynamics, and constraints," *Automatica*, vol. 35, no. 3, 1999, pp. 407–427.
[2] D. Q. Mayne, and S. Raković, "Model predictive control of constrained piecewise affine discrete-time systems," *International Journal of Robust and Nonlinear Control*, vol. 13, no. 3-4, 2003, pp. 261–279.
[3] S.J. Qin, V. Martinez, B.A. Foss, "An interpolating model predictive control strategy with application to waste treatment plant", *Comput. Chemical Eng.* 21 (1997) S881-S886.
[4] D. Dougherty, D. Cooper, "A practical multiple model adaptive strategy for single loop model predictive control", *Control Eng. Practice*, 11(2003) 141-159.
[5] Q. Chen, L. Gao, R. A. Dougal, S. Quan, "Multiple model predictive control for a hybrid ptoron exchange membrane fuel cell system", *Journal of Power Sources* 191 (2009) 473-482.
[6] C.R. Porfirio, E. Almeida Neto, D. Odloak, "Multi-model predictive control of an industrial C3/C4 splitter", *Control Eng. Practice*, 11 (2009) 765-779.
[7] P.J. van Overloop, S. Weijs, S. Dijkstra, "Multiple model predictive control on a drainage canal system", *Control Eng. Practice*, 16 (2008) 531-540.
[8] W.J. Rugh, J.S. Shamma, "A survey if research on gain scheduling", *Automatica*, 36 (10) 2000 1401-1425.
[9] F.J. Doyle, H.P. Kwatra, J.S. Schwaber, "Dynamic gain scheduled process control", *Chemical Eng. Sci.* 53(15) (1998) 2675-2690.
[10] K. Schott, B.W. Bequette, "Control of chemical reactors using multiple-model adaptive control", *in IFAC DYCORD 95*, Helsingor, Denmark 1005, pp 345-350.
[11] J.A. Rodriguez, J.A. Romagnoli, G. Goodwin, "Supervisory multiple regime control" *J. of Process Control* 13 (2003) 177-190.
[12] B. Pregelj, S. Gerkšič, and S. Strmčnik, "Pattern recognition-based supervision of indirect adaptation for better disturbance handling," *ISA Transactions*, 46, (4) 2007 561-568.
[13] E. N. Pistikopoulos, V. Dua, N. A. Bozinis, A. Bemporad, and M. Morari, "On-line optimization via off-line parametric optimization tools," *Computers and Chemical Engineering*, vol. 24, 2000, pp. 183–188.
[14] A. Bemporad, M. Morari, V. Dua, and E. N. Pistikopoulos, "The explicit linear quadratic regulator for constrained systems," *Automatica*, vol. 38, no. 1, 2002, pp. 3–20.
[15] V. Sakizlis, V. Dua, J. D. Perkins, and E. N. Pistikopoulos, "The explicit control law for hybrid systems via parametric programming", in *2002 Proc. American Control Conference*, Anchorage, vol. 1, pp. 674–679.
[16] F. Borrelli, *Constrained optimal control of linear and hybrid systems*. Lecture notes in control and information sciences (vol. 290), Berlin: Springer, 2003.
[17] P. Grieder, M. Kvasnica, M. Baotić, M. Morari, "Stabilizing low complexity feedback control of constrained piecewise affine systems," *Automatica*, vol. 41, no. 10, Oct. 2005, pp. 1683–1694.
[18] A. Bemporad, N. Giorgetti, "Logic-based solution methods for optimal control of hybrid systems," *IEEE Transactions on Automatic Control*, vol. 51, no. 6, Jun. 2006, pp. 963–976.
[19] M. Baotić, F. J. Christophersen, and M. Morari, "Constrained optimal control of hybrid systems with a linear performance index," *IEEE Transactions on Automatic Control*, vol. 51, Dec. 2006, pp. 1903–1919.
[20] K. Kouramas, and V. Dua, "Hybrid parametric model-based control," in *Multi-Parametric Model-Based Control*, E. N. Pistikopoulos, M. C.. Georgiadis, and V. Dua, Ed., Weinheim: Wiley-VCH, 2007, pp. 25–48.
[21] M. Barić, P. Grieder, M. Baotić, and M. Morari, "An efficient algorithm for optimal control of PWA systems with polyhedral performance indices," *Automatica*, vol. 44, 2008, pp. 296–301.
[22] *POP, Parametric optimization software*. ParOS Parametric Optimisation Solutions Ltd, 2003.
[23] M. Kvasnica, P. Grieder, and M. Baotić. (2005) Multi-parametric toolbox [Online]. Available: http://control.ee.ethz.ch/ mpt/
[24] A. Bemporad, "Hybrid toolbox for real-time applications, user's guide". University of Siena, Technical report, 2006.
[25] Dua, V., E. N. Pistikopoulos, "An algorithm for the solution of multiparametric mixed integer linear programming problems," *Annals of Operations Research*, vol. 99, 2000, pp. 123–139.
[26] V. Dua, N. A. Bozinis, and E. N. Pistikopoulos, "A multiparametric programming approach for mixed integer and quadratic engineering problems," *Computers and Chemical Engineering*, vol. 26, 2002, pp. 715–733.
[27] N. P. Faisca, V. Dua, and E. N.Pistikopoulos, "Multiparametric linear and quadratic programming," in *Multi-Parametric Model-Based Control*, E. N. Pistikopoulos, M. C.. Georgiadis, and V. Dua, Ed., Weinheim: Wiley-VCH, 2007, pp. 3–23.
[28] T. A. Johansen, W. Jackson, R. Schreiber, and P. Tøndel, "Hardware synthesis of explicit model predictive controllers," *IEEE Transactions on Control Systems Technology*, vol. 15, 2007, pp. 191–197.
[29] K. R. Muske, and T. A. Badgwell, "Disturbance modeling for offset-free linear model predictive control," *Journal of Process Control*, vol. 12, 2002, pp. 617–632.
[30] G. Pannocchia, and J. B. Rawlings, "Disturbance models for offset-free model predictive control," *AIChE Journal*, vol. 49, 2003, pp. 426–437.
[31] Sakizlis, V. (2003). Design of model based controllers via parametric programming. PhD thesis, Imperial College, London.
[32] Sakizlis, V., Dua, V., Perkins, J. D., and Pistikopoulos, E. N. (2004). Robust model-based tracking control using parametric programming. Computers and Chemical Engineering, 28:195–207.
[33] M. Vašak, M. Baotić, M. Morari, I. Petrović, and N. Perić, "Constrained optimal control of an electronic throttle," *International Journal of Control*, vol. 79, no. 5, 2006, pp. 465–478.
[34] G. Ferrari-Trecate, D. Mignone, and M. Morari, "Moving horizon estimation for hybrid systems," *IEEE Transactions on Automatic Control*, vol. 47, no. 10, 2002, pp. 1663–1676.
[35] G. Böker, and J. Lunze, "Stability and performance of switching Kalman filters," *International Journal of Control*, vol. 75, no. 16, 2002, pp. 1269–1281.
[36] A. Alessandri, and P. Coletta, "Design of observers for switched discrete-time linear systems," *2003 Proc. American Control Conference*, Denver CO, pp. 2785–2790.
[37] L. Pina, and M. A. Botto, "Simultaneous state and input estimation of hybrid systems with unknown inputs,', *Automatica*, vol. 42, no. 5, 2006, pp. 755–762.
[38] J. M. Maciejowski, *Predictive Control with Constraints*, Harlow UK: Prentice Hall, 2002.
[39] F. D. Torrisi, and A. Bemporad, "HYSDEL — A tool for generating computational hybrid models," *IEEE Transactions on Control Systems Technology*, vol. 12, no. 2, 2004, 235–249.
[40] L. Özkan, and M. V. Kothare, "Stability Analysis of a Multi-Model Predictive Control algorithm with Application to Control of Chemical Reactors," *Journal of Process Control*, vol. 16, no. 2, Feb. 2006, pp. 81–90.
[41] S. Gerkšič, S. Strmčnik, and T. J. J. van den Boom, "Feedback action in predictive control: an experimental case study," *Control Engineering Practice*, vol. 16, 2008, pp. 321–332.
[42] S. Gerkšič, B. Pregelj, "Disturbance rejection tuning of a tracking multi-parametric predictive controller," *2009 IEEE International Conference on Industrial Technology*, submitted for publication.
[43] A. Grancharova T. A. Johansen, and J. Kocijan, "Explicit model predictive control of gas-liquid separation plant via orthogonal search tree partitioning," *Computers and Chemical Engineering*, vol. 28, 2004, pp. 2481–2491.
[44] B. Pregelj and S. Gerkšič, "Hybrid Multi-Parametric Model Predictive Control of a Nonlinear Process Approximated with a Piecewise Affine Model," in *Proc of ECC09*, Budapest, 2009.